



AD FALCON API Manual

Analytical and Finite Element Solutions for Undrained Expansion of Cylindrical Cavities in Clay

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1 Analytical and Finite Element Solutions for Undrained Expansion of Cylindrical Cavities in Clay

1.1 File Name

fem_cavity_MCC.txt

1.2 Problem Description

This study examines the **undrained expansion of cylindrical cavities** in clay using both **analytical solutions** and **finite element analysis (FEA)**. The **modified Cam-Clay (MCC) model** is used to simulate the soil behavior.

The **analytical solutions** for undrained cavity expansion in soils modeled by the **modified Cam-Clay model** were derived by **Collins and Yu (1996)**.

Reference: > Collins, I.F. and Yu, H.S., 1996. *Undrained cavity expansions in critical state soils*. International Journal for Numerical and Analytical Methods in Geomechanics, 20(7), pp.489-516.

These solutions provide a benchmark for verifying the accuracy of finite element simulations.

1.3 Finite Element Model Setup

- The analytical solutions of **Collins and Yu (1996)** assume an **infinite soil domain**.
- In the **finite element analysis**, a **finite outer boundary** is set at **100 times the initial cavity radius**.

1.4 Loading and Boundary Conditions

- The **total radial pressure** at the cavity wall is **increased gradually**, causing cavity expansion.
- The cavity expands by **50%**, reaching a **final radius of 1.5 m**.
- Expansion occurs over **100 increments**.
- **All boundaries are sealed** to prevent drainage.
- The mesh consists of **6-noded triangular elements**, with **pore pressure degrees of freedom** at the **three corner nodes** of each element.

1.5 Soil Parameters

The soil parameters correspond to **London Clay**:

- **Critical State Parameter (M):** $M = 0.888$
- **Compressibility Index λ :** 0.161
- **Swelling Index κ :** 0.062

- **Specific Volume of Normally Consolidated Line (NCL) at Unit Pressure** v_N : 2.8276
 - **Specific Volume of Critical State Line (CSL) at Unit Pressure** v_{csl} : 2.759
- Note that we have the following relationship between v_{csl} and v_N :

$$v_N = v_{csl} + (\lambda - \kappa) \ln 2 \quad (1)$$

The **initial void ratio** and **overconsolidation ratio (OCR)** are:

- **Initial Void Ratio** e_0 : 1.0
- **Overconsolidation Ratio (OCR)**: 1.0

The **initial preconsolidation pressure** p_0^0 is determined as:

$$p_0^0 = \exp\left(\frac{v_N - e_0 - 1}{\lambda}\right) = 170.8 \quad (2)$$

Assuming an **isotropic initial stress state** with **OCR = 1**, the initial stresses are:

$$\sigma_{z0} = \sigma_{r0} = \sigma_{h0} = 170.8 \quad (3)$$

Normalization of Stresses In the analytical solutions of **Collins and Yu (1996)**, all stresses are **normalized by the undrained shear strength** S_u , which is given by:

$$S_u = 0.5M \exp\left(\frac{v_{csl} - e_0 - 1}{\lambda}\right) = 49.52 \quad (4)$$

Results The comparison of results between **pore water pressure** and **total radial stress evolution** (at **0,1**), normalized by S_u , are shown in Figures 1 and 2 alongside the **analytical solution proposed by Yu and Collins**, demonstrating good agreement.

Figure 1: Pore Water Pressure vs. Expansion

Figure 2: Total Radial Stress vs. Expansion

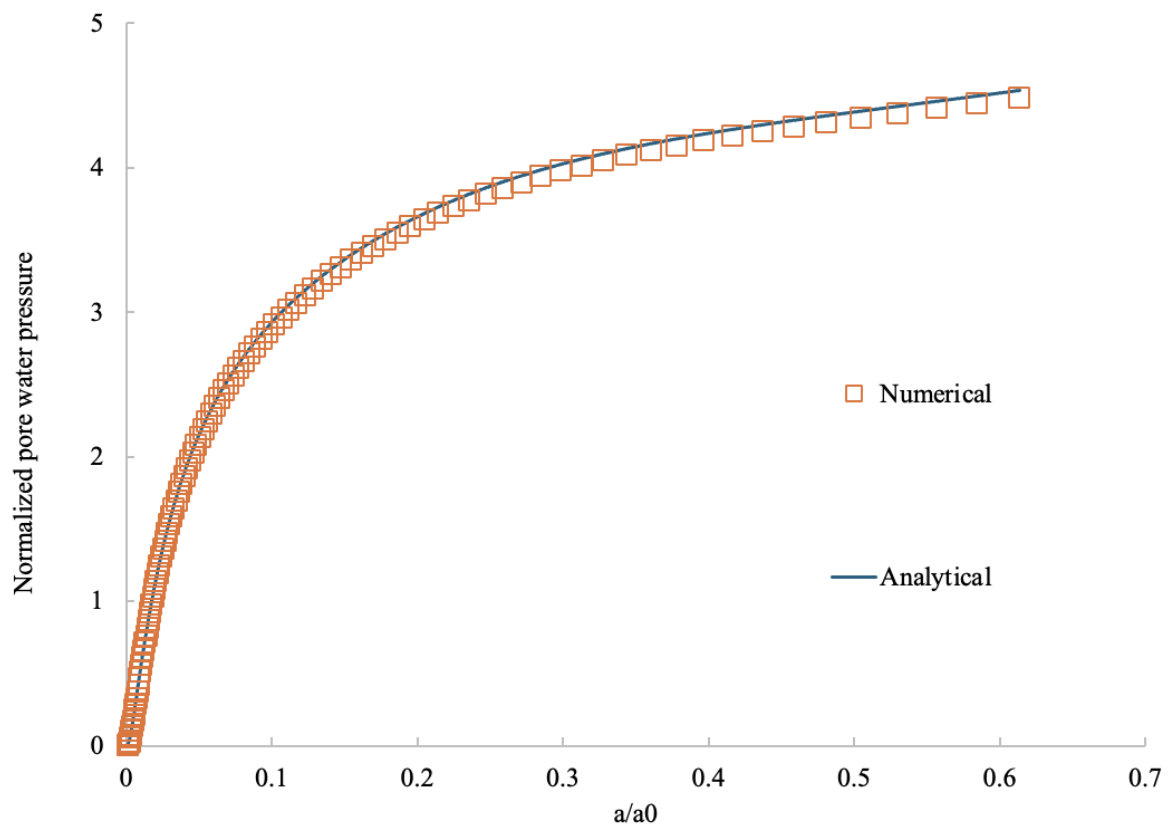


Figure 1: figure1 ...

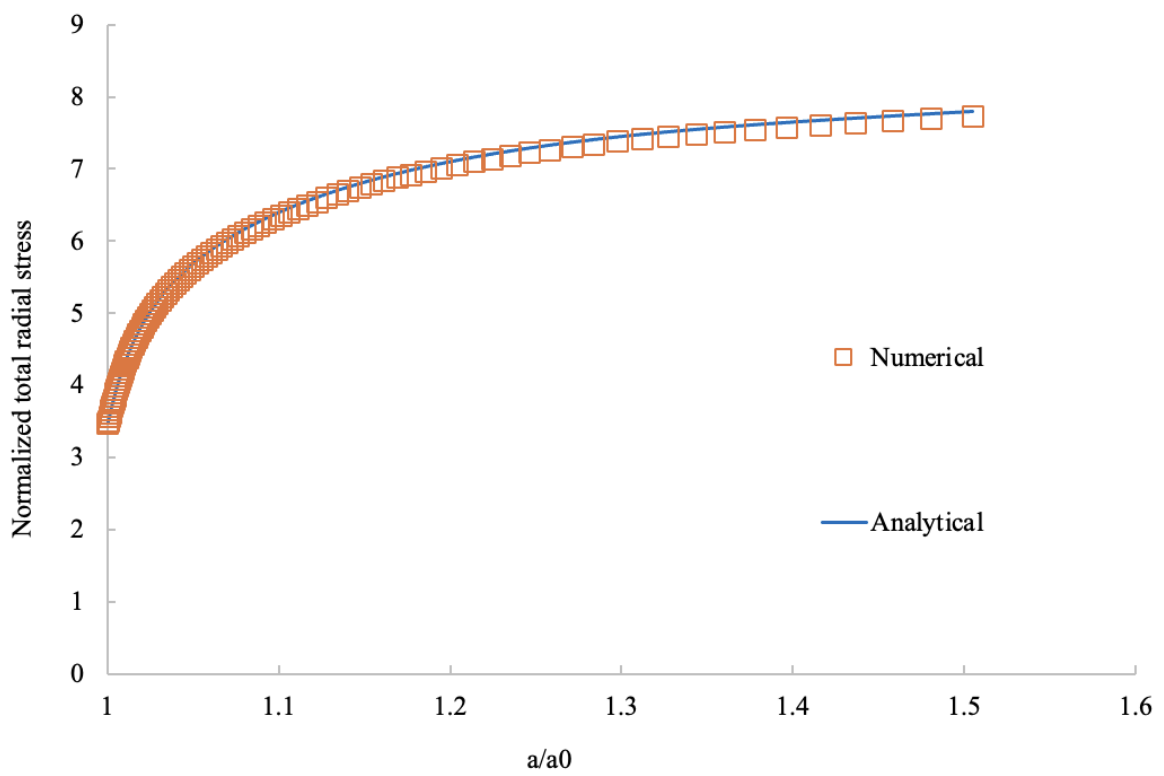


Figure 2: figure2 ...