AD FALCON API Manual

Subloading Surface Model

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1 Subloading Surface Model

The `SSLSMModel` implements the sub/super-loading surface framework of Zhao et al. (2005). A Cam-Clay-type reference surface is combined with a subloading ratio `R` and a structure ratio `RS` (denoted R_n in Zhao et al.), allowing plastic straining to occur smoothly before the normal surface is reached.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/SSLSMModel.cpp path/to/SSLSMModel.hpp Mechanical \
    Phi=35.3 Lambda=0.05 Kappa=0.035 Nu=0.3 UseLambdaStar=1 \
    Mmax=1.43 v_N=2.17 m_sub=0.127 a_sub=0.092 Ad_sub=0.95 \
    P_min=1.0 STOL=1e-7 FTOL=1e-6 LTOL=1e-6 \
    CustomVariable=IsotropicHardening,R,RS
```

For readability the example is wrapped across multiple lines; in input files the full `@UMAT:` directive should be written on a single line.

1.2 Material Parameters

Symbol	Keyword in input	Required	Meaning
ϕ	`Phi`	✓	Critical-state friction angle.
λ	`Lambda`	✓	Virgin compression index. If `UseLambdaStar=1`, interpreted as λ^*.
κ	`Kappa`	✓	Swelling / reloading index.
ν	`Nu`	✓	Poisson ratio.

Symbol	Keyword in input	Required	Meaning
v_N	v_N	✓	Specific volume at $p = 1$ on the normal compression line.
$M_{\max}$	Mmax	✓	Maximum critical-state slope used in the Lode-angle function.
m	m_sub	✓	Evolution rate of the subloading ratio R.
a_{sub}	a_sub	✓	Evolution rate of the structure ratio R_n.
A_d	Ad_sub	✓	Weight in the hardening measure $d\varepsilon_d$.
$P_{\min}$	P_min	✓	Minimum pressure used in elastic stiffness evaluation.
–	UseLambdaStar	×	If 1, Lambda and Kappa are treated as starred compressibility indices.
–	STOL	✓	Substepping tolerance.
–	FTOL	✓	Yield tolerance.
–	LTOL	✓	Load-unload tolerance.

1.3 Custom State Variables

Declare custom variables using `CustomVariable=` in the `@UMAT:` line.

Name	Required	Meaning
`IsotropicHardening`	✓	p_c, the (normal) Cam-Clay reference size used by the SSLSM formulation.
`R`	✓	Subloading ratio $R \in (0, 1]$ controlling the active subloading surface size.
`RS`	✓	Structure / superloading similarity ratio $R_n \in (0, 1]$. This controls the superloading surface size.
`OCR`	optional	OCR-like initializer used to set p_c from the current mean effective pressure p_0.
`OCRonSuperloading`	optional	Switch for how `OCR` is interpreted in the initializer. If `0` (or omitted), OCR is defined on `pc` (`IsotropicHardening`). If `1`, OCR is defined on the superloading cap size `p_tilde_c = pc/RS`. Use `0` or `1` (any nonzero value is treated as `1`). See Initialization for the formulas.
`UpdateVoidRatio`	optional	If `1`, rebuild the initial void ratio so it is consistent with p_0 and p_c.

1.4 Sign Convention and Invariants

The code uses the usual FALCON tension-positive stress convention. The SSLSM invariant helper then defines mean effective stress as positive in compression:

$$p = -\sigma_m = -\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

and

$$q = \sqrt{3J_2}, \qquad \eta = \frac{q}{p}$$

so the constitutive equations are written in the standard Cam-Clay compression-positive form even though the stored stress tensor uses compression-negative components.

1.5 Three-Surface Framework

Zhao et al. (2005) define the similarity ratios

$$R = \frac{p}{\tilde{p}} = \frac{q}{\tilde{q}}, \qquad R_n = \frac{p^*}{\tilde{p}} = \frac{q^*}{\tilde{q}} \tag{1}$$

where:

- (p, q) is the current stress point on the subloading surface
- (p^*, q^*) is the image point on the normal surface
- $(\tilde{p}, \tilde{q})$ is the image point on the superloading surface

With homologous Cam-Clay-type surfaces,

$$\tilde{p}_c = \frac{p_c}{R_n}, \qquad p_{\mathsf{sub}} = R\tilde{p}_c = R\frac{p_c}{R_n} \tag{2}$$

and the implemented subloading yield function is

$$f_{\mathsf{sub}}(p, q) = q^2 + M(\theta)^2\, p\,(p - p_{\mathsf{sub}}) \tag{3}$$

When `R = 1`, the stress point lies on the reference surface. For `R < 1`, the active subloading surface is smaller.

1.6 Lode-Angle Dependence

The critical-state slope varies with Lode angle according to the Zhao et al. form:

$$M(\theta) = M_{\max}\left[\frac{2a_{\mathsf{Lode}}^4}{1 + a_{\mathsf{Lode}}^4 - (1 - a_{\mathsf{Lode}}^4)\sin(3\theta)}\right]^{1/4}, \qquad a_{\mathsf{Lode}} = \frac{3 - \sin\phi}{3 + \sin\phi} \tag{4}$$

This is the expression implemented in `computeM(...)`.

1.7 Elasticity and Hardening

The elastic bulk modulus is

$$K = \frac{v\,p}{\kappa} \tag{5}$$

with

$$G = \frac{3(1 - 2\nu)}{2(1 + \nu)}K \tag{6}$$

The hardening variables are $j = \{p_c, R, R_n\}$ and evolve through

$$dj = d\lambda\, B \tag{7}$$

with the coded hardening terms

$$B_1 = \frac{v\, p_c}{\lambda - \kappa} \frac{\partial f_{\text{sub}}}{\partial p}, \qquad B_2 = -\frac{v\, M}{\lambda - \kappa}\, m\, \ln(R)\, d\varepsilon_d, \qquad B_3 = \frac{v\, M\, a_{\text{sub}}}{\lambda - \kappa}\, R\,(1 - R_n)\, d\varepsilon_d \tag{8}$$

and

$$d\varepsilon_d = \sqrt{(1 - A_d)\left(\frac{\partial f_{\text{sub}}}{\partial p}\right)^2 + A_d\left(\frac{\partial f_{\text{sub}}}{\partial q}\right)^2} \tag{9}$$

The code updates `pc`, `R`, and `RS` together during the stress integration.

1.8 Initialization

Initialization is performed once at the start of an analysis to condition the provided custom variables. The SSLSM model has three similar Cam-Clay-type surfaces, which can be summarized by their cap sizes in mean effective pressure:

$$\tilde{p}_c = \frac{p_c}{R_n}, \qquad p_{\text{sub}} = R\,\tilde{p}_c = R\frac{p_c}{R_n} \tag{10}$$

where:

- p_c =`IsotropicHardening`$ is the normal/reference surface size,
- R =R$ is the subloading ratio, and
- R_n =RS$ is the structure / superloading similarity ratio.

The implemented subloading yield function is evaluated using p_{sub} (see Eq. (3) and Eq. (2)).

1.8.1 What OCR actually does

`OCR` is an *initializer* for p_c based on the current mean effective pressure p_0 computed from the provided initial stress field:

$$p_0 = \max(P_{\min}, p(\boldsymbol{\sigma}_0)) \tag{11}$$

Two common interpretations of "OCR" are supported:

1. OCR defined on the normal/reference surface (default, `OCRonSuperloading=0`)

$$\text{OCR} = \frac{p_c}{p_0} \quad \Rightarrow \quad p_c = \text{OCR}\, p_0 \tag{12a}$$

2. OCR defined on the superloading surface (`OCRonSuperloading=1`)

$$\text{OCR} = \frac{\tilde{p}_c}{p_0} = \frac{p_c/R_n}{p_0} \quad \Rightarrow \quad p_c = R_n \, \text{OCR} \, p_0 \tag{12b}$$

This second option is useful if you want the superloading cap size to correspond to the OCR you specify, i.e.
$tildep_c = \text{OCR}\, p_0$.

If `OCRonSuperloading` is omitted, it defaults to `0`. Use `0` or `1` (any nonzero value is treated as `1`).

There is no inconsistency when `OCRonSuperloading=0`. It only means that the number you provide as `OCR` is interpreted as p_c/p_0 rather than
$tildep_c/p_0$. Because the SSLSM model has two related cap sizes, choosing one definition implies a corresponding value for the other:

- If `OCRonSuperloading=0`: $p_c = \text{OCR}\, p_0$ and $tildep_c/p_0 = \text{OCR}/R_n$.
- If `OCRonSuperloading=1`: $tildep_c = \text{OCR}\, p_0$ and $p_c/p_0 = R_n\, \text{OCR}$.

1.8.2 Precedence / inputs you must provide

Initialization uses the following rule:

- If `OCR` is provided (and finite, > 0), it overrides any input `IsotropicHardening` value and initializes p_c using Eq. (12a) or (12b).
- If `OCR` is not provided, `IsotropicHardening` is interpreted directly as p_c.

In all cases, you must provide physically meaningful initial values:

$$p_c > 0, \qquad 0 < R \le 1, \qquad 0 < R_n \le 1 \tag{13}$$

1.8.3 Void ratio conditioning (optional)

If `UpdateVoidRatio=1`, the initial void ratio is rebuilt from the Cam-Clay compression relation

$$v(p_0, p_c) = v_N - \lambda \ln p_c + \kappa \ln\left(\frac{p_c}{p_0}\right) \tag{14}$$

so that the initial specific volume is consistent with the chosen state.

1.9 Implementation Notes

- `RS` in the code stores the paper's R_n.

- The stress update is performed in `calculateStressIncrement(...)` using explicit adaptive substepping.
- Plasticity activates when the initial `R` and `RS` are below 1 or when the loading path reaches the active surface.

1.10 FALCON mini

The packaged mini tool id is `Subloading`. It lives under `mini_tools/Subloading`.

1.10.1 How to run

Run any packaged analysis by passing its case directory:

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool
Subloading --mini-input
/path/to/UMATLIB_FALCON/falcon_minis/Subloading/cases/drained
```

Packaged simulation families:

Packaged case	Path	Purpose
Drained triaxial	`cases/drained/input.txt`	Drained triaxial reference path at constant radial stress.
Drained OCR sweep	`cases/drained_ocr_sweep/input.txt`	Multi-case drained sweep showing OCR sensitivity under the same triaxial loading path.
Drained structure sweep	`cases/drained_rs_sweep/input.txt`	Multi-case drained sweep showing sensitivity to the initial superloading ratio `RS`.
Undrained triaxial	`cases/undrained/input.txt`	Undrained constant-volume triaxial reference path.
Isotropic strain-controlled	`cases/isotropic_strain/input.txt`	Isotropic compression/unloading/reloading reference path.
Isotropic constant mean stress	`cases/isotropic_const_p/input.txt`	Isotropic path with controlled mean stress.

1.10.2 Input syntax

`input.txt` uses whitespace-delimited `Key Value` pairs, one item per line, for example:

```
Mode Undrained
Phi 30.0
Lambda 0.15
OCR 1.5
R 0.5
RS 0.75
```

The main selector is `Mode`.

`Mode` value	Meaning in the standalone mini	Mechanical constraint
`DrainedConstSigmaR`	Axisymmetric drained triaxial loading.	Radial stress is kept approximately constant.
`Undrained`	Axisymmetric undrained triaxial loading.	Zero total volumetric strain.
`IsotropicStrain`	Isotropic strain-controlled loading.	Prescribed isotropic strain history.
`IsotropicConstP`	Isotropic loading at controlled mean stress.	Mean stress target enforced by the standalone driver.

Mini inputs used by the packaged cases:
Constitutive parameters:

Input key	Used by	Required / choices / defaults	Meaning
`Phi`, `Lambda`, `Kappa`, `Nu`	all cases	Required in packaged cases	Core SSLSM constitutive parameters.
`UseLambdaStar`	optional	Optional; choices `0` / `1`; driver default if omitted	Switch for the compression formulation.
`Mmax`, `m_sub`, `a_sub`, `Ad_sub`	all cases	Required in packaged cases	Subloading and superloading controls governing the yield surface and structure evolution.
`v_N`	all cases	Required in packaged cases	Specific-volume intercept.
`P_min`	all cases	Optional; driver default if omitted	Pressure floor used in the tangent evaluation.

Input key	Used by	Required / choices / defaults	Meaning
`STOL`, `FTOL`, `LTOL`	all cases	Optional; packaged cases may set them explicitly	Integration and branching tolerances.

Initial-state inputs:

Input key	Used by	Required / choices / defaults	Meaning
`p0`	all cases	Required in packaged cases	Initial mean stress magnitude used to build the starting stress state.
`VoidRatio`	all cases	Required in packaged cases	Initial void ratio.
`OCR`	all cases	Required in packaged cases	Overconsolidation measure used to initialize the hardening size.
`R`, `RS`	all cases	Required in packaged cases	Initial subloading and superloading ratios.
`OCRonSuperloading`	optional	Optional; choices `0` / `1`; driver default if omitted	Switch controlling how OCR is interpreted in the initializer.

There is no separate `mpre` keyword in the SSLSM input. The initial preconsolidation size is derived during initialization from `p0`, `OCR`, `R`, `RS`, and optional `OCRonSuperloading`, as described in Eq. (11) below.

The standalone mini also supports numbered sweep keys such as `OCR_1`, `OCR_2`, `R_1`, and `RS_1`. When those are used, the driver writes one CSV per case combination instead of a single `sslsm_results.csv`.

Loading controls:

Input key	Used by	Required / choices / defaults	Meaning
`dEpsAxial`, `nSteps`	drained and undrained	Required for triaxial modes	Axial strain increment and number of triaxial loading steps.

`dEpsIso`, `nSteps`
`Load`, `nSteps`
`Unload`, `nSteps`
`Reload`
	isotropic strain-controlled	Required for `Mode = IsotropicStrain`	Isotropic strain increment and step counts for loading branches.

`pStart`, `pEnd`	isotropic constant mean stress	Required for `Mode = IsotropicConstP`	Mean-stress targets for the isotropic constant-`p` path.

1.10.3 Hydromechanical assumptions

The packaged SSLSM mini is saturated only:

- there is no suction or retention update
- the state evolution is controlled by the subloading ratio `R`, the superloading ratio `RS`, the hardening size `pc`, and the void ratio `e`
- the main point of the mini is to show how structured and overconsolidated states evolve under different mechanical constraints

1.10.4 Sample input

Drained triaxial example Path: `mini_tools/Subloading/cases/drained/input.txt`

```
Mode DrainedConstSigmaR
Phi 35.3
Lambda 0.05
Kappa 0.035
Mmax 1.43
Nu 0.3
v_N 2.17
m_sub 0.3
p0 34.5
VoidRatio 0.82
dEpsAxial -1.0e-4
nSteps 2500
OCR 5.0
R 0.5
RS 0.75
```

This packaged case starts from a genuine subloading state with `R < 1` and `RS < 1`, so the internal state variables evolve visibly during drained shearing.

Undrained triaxial example Path: `mini_tools/Subloading/cases/undrained/input.txt`

This packaged case uses the same type of explicitly prescribed subloading state but switches the triaxial constraint to constant volume. It is the direct companion path for comparing drained and undrained evolution of `R`, `RS`, and `pc`.

Drained OCR sweep example Path: `mini_tools/Subloading/cases/drained_ocr_sweep/input.txt`

This packaged sweep uses `OCR_1`, `OCR_2`, and `OCR_3` to generate three drained triaxial runs from the same base material state. It is the most compact way to compare normally consolidated and overconsolidated responses without duplicating the rest of the input deck.

Drained structure-ratio sweep example Path: `mini_tools/Subloading/cases/drained_rs_sweep/input.txt`

This packaged sweep uses `RS_1`, `RS_2`, and `RS_3` to compare how the initial superloading ratio changes the drained response while keeping `OCR`, `R`, and the loading path fixed.

Isotropic examples Paths:

- `mini_tools/Subloading/cases/isotropic_strain/input.txt`
- `mini_tools/Subloading/cases/isotropic_const_p/input.txt`

These packaged cases isolate the isotropic hardening and subloading-surface evolution without deviatoric triaxial effects.

1.10.5 Output files and columns

All packaged cases write `sslsm_results.csv`.

Output file	Produced by	Main use
`sslsm_results.csv`	all cases	Main mechanical and state-variable history for the packaged SSLSM cases.

Primary columns:

Output column	Meaning
`step`	Load-step index.

Output column	Meaning
`exx`, `eyy`, `ezz`, `sxx`, `syy`, `szz`	Normal strain and stress components.
`q`, `sigma_m`	Deviatoric stress and mean stress measure used by the standalone driver.
`R`, `RS`, `pc`, `e`	Core SSLSM state variables: subloading ratio, superloading ratio, hardening size, and void ratio.

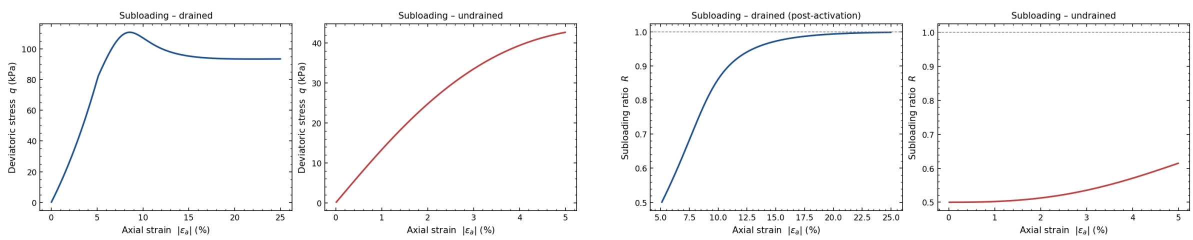
The drained and undrained packaged examples both start from explicit subloading states so the internal variables evolve measurably during the run.

Sweep cases write one CSV per generated combination, for example `sslsm_results_OCR5.000_Rn0.500_RS0.750.csv`.

The plots in the next section are generated from these packaged case CSVs.

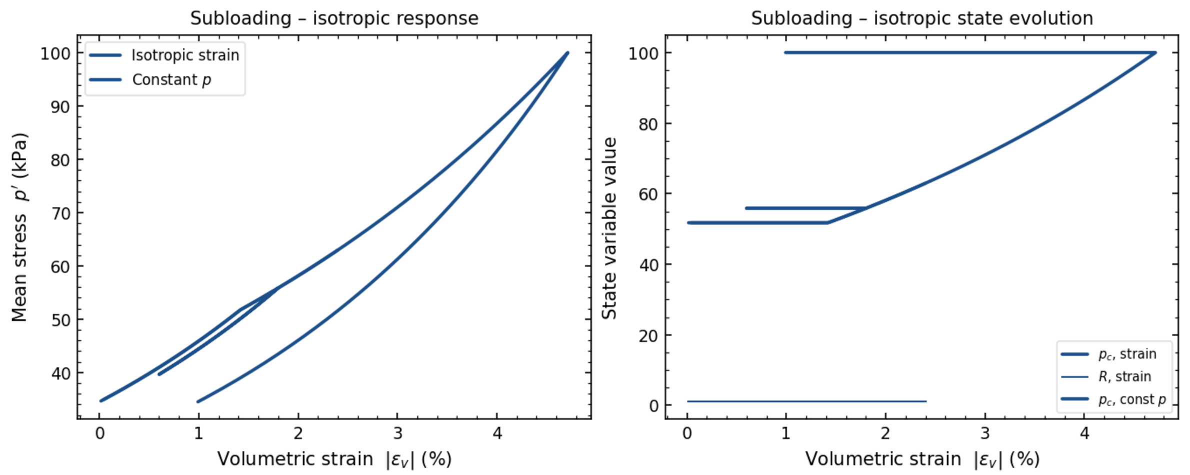
1.11 Results

The plots below are produced directly from the bundled FALCON mini case inputs under `mini_tools/Subloading/cases`. The packaged examples use the same structured initial-state idea for both drained and undrained paths so the evolution of the subloading ratio can be read cleanly from the figures.



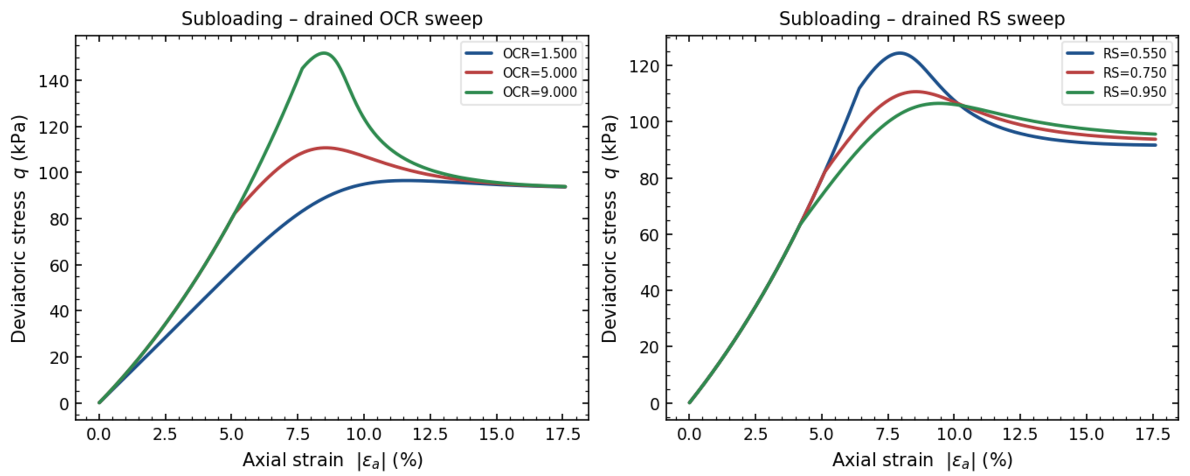
Bundled cases `cases/drained/input.txt` and `cases/undrained/input.txt`. Left: drained and undrained triaxial stress-strain responses on separate axes so the smaller-strain undrained path remains visible. Right: the corresponding evolution of the subloading ratio `R`.

1.11.1 Isotropic reference paths



Bundled cases cases/isotropic_strain/input.txt and cases/isotropic_const_p/input.txt. Left: isotropic response in mean stress versus volumetric strain. Right: evolution of the hardening size pc and the subloading ratio R along the isotropic paths.

1.11.2 Drained sensitivity sweeps



Bundled cases cases/drained_ocr_sweep/input.txt and cases/drained_rs_sweep/input.txt. Left: drained triaxial OCR sensitivity. Right: drained triaxial sensitivity to the initial superloading ratio RS.

1.12 Applications and limitations

- Best suited to structured and overconsolidated soils where subloading and superloading effects are central to the response.
- Appropriate for uncoupled and effective-stress-based coupled analyses, especially for single-point studies of OCR, R, and RS sensitivity.
- The documented model is not an intrinsic unsaturated suction-hardening law and is not aimed at cyclic mobility / liquefaction of sands.

1.13 References

1. Zhao, J., Sheng, D., Sloan, S. W., & Abbo, A. J. (2005). *Explicit stress integration of complex soil models*. International Journal for Numerical and Analytical Methods in Geomechanics, 29, 1209-1229.