



AD FALCON API Manual

Problem Definition

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Analytical Solution for the Dynamic Response of a Laterally Restrained Column under a Step Force

We consider a one-dimensional column whose dynamic behavior is governed by axial equilibrium under an applied step force.

1 Problem Definition

Geometry:

The column has a length of 10 m :

$$L = 10 \text{ m} \quad (1)$$

Material Properties:

The material properties are given by:

$$E = 2.9376 \times 10^9 \text{ Pa}, \quad \rho = 2700 \text{ kg/m}^3 \quad (2)$$

Damping:

A viscous damping term is incorporated via a damping coefficient, c . Thus, the damping factor is simply:

$$\text{damp} = c \quad (3)$$

Applied Loading and Boundary Conditions:

- Bottom (Fixed):

$$u(0, t) = 0 \quad (4)$$

- Top (Step Force):

Instead of a prescribed displacement, a step force of amplitude F_0 is applied at the top. The applied force per unit area (stress) is given by

$$\frac{F_0}{A} \quad (5)$$

The corresponding Neumann boundary condition at the top relates the strain at the top to the applied stress via Hooke's law:

$$u_x(L, t) = \frac{F_0}{EA} \quad (6)$$

In finite difference form, this condition is approximated as:

$$\frac{u(N-1) - u(N-2)}{dx} = \frac{F_0}{EA} \quad (7)$$

which can be rearranged to:

$$u(N-1) = u(N-2) + dx \cdot \frac{F_0}{EA} \quad (8)$$

Governing Equation

The dynamic behavior of the column is described by the equation of motion:

$$\rho A u_{tt}(x, t) + c u_t(x, t) = EA u_{xx}(x, t), \quad 0 \leq x \leq L \quad (9)$$

See the finite-element counterpart and notation in [Uncoupled Analysis](#), including the mass, damping and stiffness terms. At static equilibrium (after transients decay), the strain at the top satisfies

$$u_x(L) = \frac{F_0}{EA} \Rightarrow u(L) = \frac{F_0}{EA} L \quad (10)$$

Finite Difference Approximation

The domain is discretized in space and time:

- Spatial Discretization:

The domain $0 \leq x \leq L$ is divided into $N = 101$ nodes with spacing

$$dx = \frac{L}{N - 1} \quad (11)$$

- Temporal Discretization:

The time step dt is chosen to satisfy the CFL condition (i.e., $dt \leq \frac{dx}{\sqrt{D}}$ with $D = \frac{E}{\rho}$), and the total simulation time is set to T_{final} .

The finite difference approximations used are:

- Second Time Derivative:

$$u_{tt} \approx \frac{u_{n+1} - 2u_n + u_{n-1}}{dt^2} \quad (12)$$

- First Time Derivative:

$$u_t \approx \frac{u_n - u_{n-1}}{dt} \quad (13)$$

- Second Spatial Derivative:

$$u_{xx} \approx \frac{u[i + 1] - 2u[i] + u[i - 1]}{dx^2} \quad (14)$$

Thus, the update formula for interior nodes becomes:

$$u_{n+1}[i] = 2u_n[i] - u_{n-1}[i] + dt^2 \left(D u_{xx}[i] - \text{damp} \frac{u_n[i] - u_{n-1}[i]}{dt} \right) \quad (15)$$

For FE time integration in FALCON, see [Time Integration Using the Newmark Method](#). Viscous damping in the FE setting corresponds to the [Damping Matrix](#). **## Python Code Implementation**

Below is the Python code that implements the finite-difference scheme for this problem:

```

# -----
# Parameters
# -----
L = 10.0          # Column length (m)
width = 1.0       # Column width (m) (assumed square cross-section)
A = width**2     # Cross-sectional area (m^2)
E = 29376000000.0 # Young's modulus (Pa)
rho = 2700       # Density (kg/m^3)
c = 0.0         # Damping coefficient (N-s/m) (assumed per unit length)
Fo = -1000.0    # Step force amplitude (N)

# Derived parameters
D = E / rho      # Wave speed squared (m^2/s^2)
damp = c        # Damping factor in units of 1/s

# -----
# Spatial and temporal discretization
# -----
N = 101         # Number of spatial nodes
dx = L / (N - 1) # Spatial step size
# CFL condition: dt <= dx/ sqrt(D)
dt = 1e-5      # Time step (s); choose dt small enough for
stability
Tfinal = 0.2   # Total simulation time (s)
Nt = int(Tfinal/dt) + 1 # Number of time steps

# -----
# Initialize displacement arrays
# -----
# We'll store displacement at three time levels: previous, current, and
next.
u_prev = np.zeros(N) # u at time n-1
u_curr = np.zeros(N) # u at time n
u_next = np.zeros(N) # u at time n+1

# For recording the displacement at the top (x = L) vs. time
time_history = [0.0]
top_disp = [u_curr[-1]]

# -----
# Time stepping loop using central differences in time
# -----
# The PDE in nondimensional form becomes:
#  $u_{tt} + \text{damp} * u_t = D * u_{xx}$ .
#

```

```

# Using finite differences:
#  $u_{tt} \approx (u_{next} - 2*u_{curr} + u_{prev}) / dt^2$ ,
#  $u_t \approx (u_{curr} - u_{prev}) / dt$ ,
#  $u_{xx}$  at interior node  $i \approx (u[i+1] - 2*u[i] + u[i-1]) / dx^2$ .
#
# Solve for  $u_{next}$ :
#  $u_{next} = 2*u_{curr} - u_{prev} + dt^2 * (D*(u_{xx}) - damp*(u_{curr} - u_{prev})/dt)$ .
#
for n in range(1, Nt):
    t = n * dt

    # Update interior nodes (i = 1 to N-2)
    for i in range(1, N-1):
        u_xx = (u_curr[i+1] - 2*u_curr[i] + u_curr[i-1]) / dx**2
        u_next[i] = (2*u_curr[i] - u_prev[i] +
                    dt**2 * (D * u_xx - damp * (u_curr[i] - u_prev[i]) /
dt))

    # Boundary conditions:
    # Bottom:  $u(0,t) = 0$ 
    u_next[0] = 0.0

    # Top: Neumann condition (stress) with step force applied for  $t \geq 0$ :
    # Approximate  $u_x(L,t) \approx (u[N-1] - u[N-2]) / dx = F_0/(E*A)$ 
    u_next[-1] = u_next[-2] + dx * F_0/(E*A)

    # Update time levels: shift the time history forward
    u_prev, u_curr = u_curr.copy(), u_next.copy()

    # Record the top displacement
    time_history.append(t)
    top_disp.append(u_curr[-1])

# -----
# Plotting the top displacement vs. time
# -----
plt.figure(figsize=(10, 6))
plt.plot(time_history, top_disp, label='Top Displacement')
plt.xlabel('Time (s)')
plt.ylabel('Displacement (m)')
plt.title('Top Displacement vs Time')
plt.grid(True)
plt.legend()

```

```
# Save the figure as a 300 dpi TIFF file
plt.savefig('top_disp.tiff', format='tiff', dpi=300)

plt.show()
```

1.1 Example Case 1: Undamped System

- **Input File:**

[soildynamics_case1.txt](#)

- **Parameters:**

```
L      = 10.0           # Column length (m)
width  = 1.0           # Column width (m) (square cross-section)
A      = width**2      # Cross-sectional area (m^2) = 1.0 m^2
E      = 2937600000.0  # Young's modulus (Pa)
rho    = 2700          # Density (kg/m^3)
c      = 0.0           # Damping coefficient (N·s/m) (no damping)
F0     = -1000.0       # Step force amplitude (N)
```

- **Discussion:**

The static solution (as $t \rightarrow \infty$) for the column under a step force is given by:

$$u_{\text{static}}(L) = \frac{F_0}{EA} L. \quad (16)$$

Substituting the parameters:

- $F_0 = -1000$ N
- $E = 2.9376 \times 10^9$ Pa
- $A = 1.0$ m²
- $L = 10.0$ m

we have:

$$u_{\text{static}}(L) = \frac{-1000 \times 10.0}{2.9376 \times 10^9 \times 1.0} = \frac{-10000}{2.9376 \times 10^9} \approx -3.40 \times 10^{-6} \text{ m} \quad (17)$$

This small negative displacement (approximately $-3.4 \mu\text{m}$) indicates a slight compression under the applied step force. Since $c = 0$, there is no energy dissipation, and the system will oscillate indefinitely about this equilibrium with a frequency determined by the wave speed $\sqrt{E/\rho}$.

- **Results:**

The plot of the top displacement $u(L, t)$ vs. time shows sustained oscillations about the

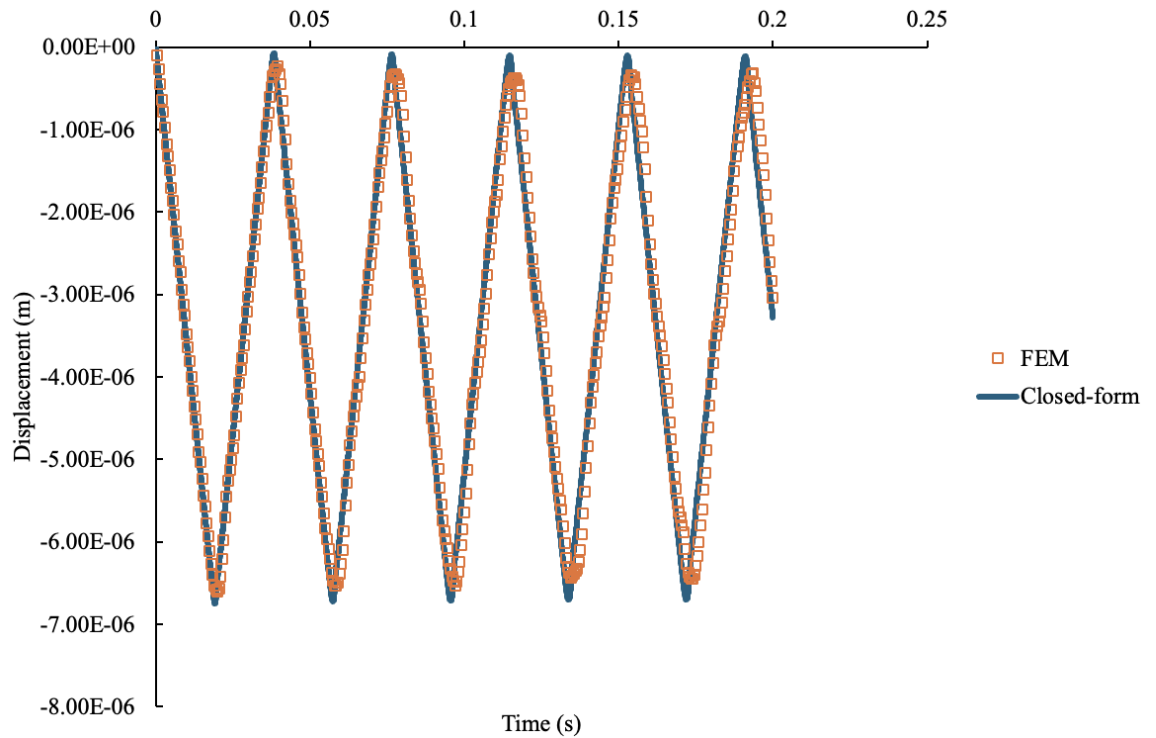


Figure 1 shows excellent agreement between the numerical simulation (markers) and the analytical prediction (line).

1.2 Example Case 2: Damped System

- **Input File:**

[soildynamics_case2.txt](#)

- **Parameters:**

All parameters are as in Case 1, except:

$$c = 10.0 \text{ (viscous damping coefficient)} \quad (18)$$

- **Discussion:**

With $c = 10.0$, the system experiences viscous damping. The static equilibrium remains:

$$u_{\text{static}}(L) = \frac{F_0}{EA} L \quad (19)$$

but the oscillations about this equilibrium decay over time. The damping term reduces the amplitude of free vibrations and accelerates convergence to the static state.

- **Results:**

The plot of $u(L, t)$ vs. time illustrates the damping effect, showing a gradual decay of

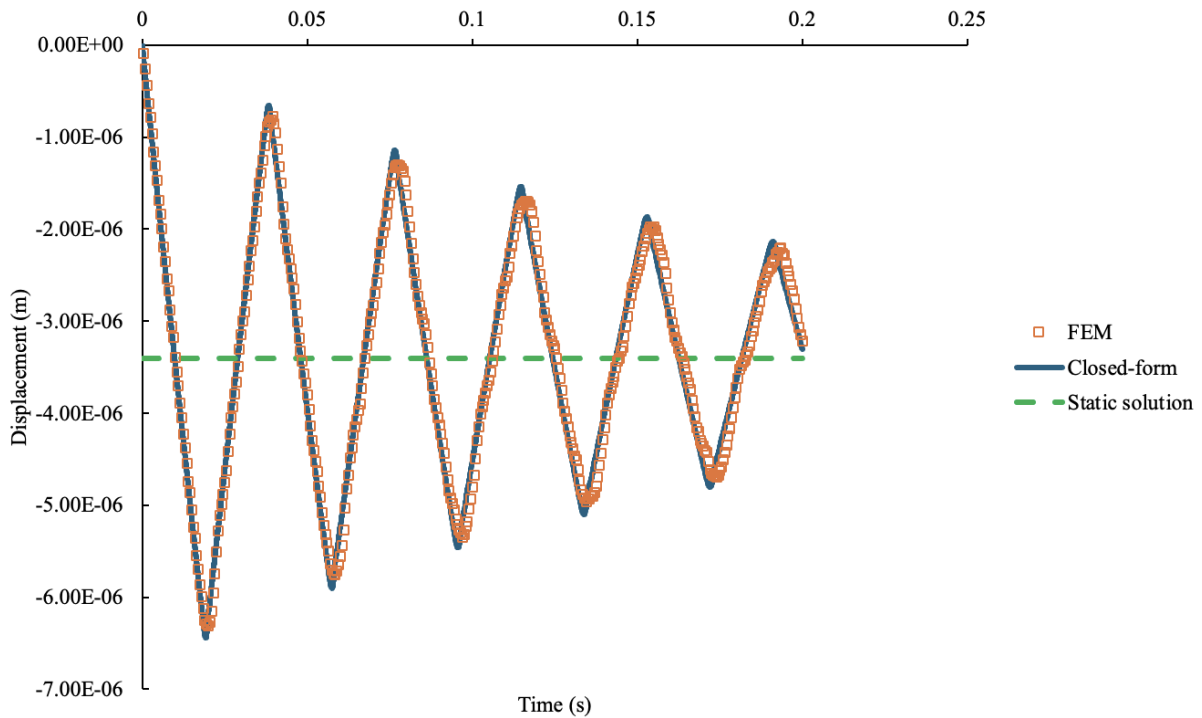


Figure 1: Figure 2: Top displacement vs time (damped)

oscillations toward the equilibrium displacement:

Figure 2 demonstrates that damping not only reduces the amplitude of oscillations but also leads to a faster settling time.

1.3 Summary of Observations

1. Undamped Case (Case 1):

- **Parameters:** $c = 0$ (no damping).
- **Behavior:** The system oscillates indefinitely around

$$u_{\text{static}}(L) \approx -3.40 \times 10^{-6} \text{ m} \quad (20)$$

2. Damped Case (Case 2):

- **Parameters:** $c = 10.0$ (viscous damping).
- **Behavior:** The same static displacement applies, but oscillations decay over time, leading to a faster and smoother convergence to the equilibrium state.

In both cases, the numerical solutions closely match the analytical predictions, confirming that the finite difference method effectively captures the dynamic response of the column under a step force.

