AD FALCON API Manual

Perfectly Matched Layer (PML) — Absorbing Truncation for Wave Problems

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1 Perfectly Matched Layer (PML) — Absorbing Truncation for Wave Problems

A **Perfectly Matched Layer (PML)** reduces spurious reflections from truncated model boundaries by modifying the governing equations inside a thin outer layer of *existing mesh elements*. In FALCON, the implementation follows a **convolutional PML (CPML)** form: spatial derivatives are scaled and augmented by **Gauss-point memory variables**.

PML is conceptually different from:

- the Sponge Layer, which adds tapered mass-proportional damping but does not modify spatial derivatives, and
- Infinite Elements, which extend the far-field geometry.

1.1 Governing idea (CPML)

In the frequency domain, classical CPML is described by a complex coordinate stretching. For the x direction,

$$\frac{\partial}{\partial \tilde{x}} = \frac{1}{s_x(\omega)}\frac{\partial}{\partial x}, \qquad s_x(\omega) = \kappa_x + \frac{\sigma_x}{\alpha_x + i\omega}. \tag{1}$$

Here $\tilde{x}$ denotes the stretched (complex) coordinate used inside the PML. A common definition is

$$\tilde{x}(x;\omega) = \int_0^x s_x(\xi,\omega)\, d\xi, \qquad d\tilde{x} = s_x\, dx.$$

In the physical domain (non-PML), $\sigma_x = 0$ and $\kappa_x = 1$ so $s_x = 1$ and (up to a constant of integration) $\tilde{x} = x$.

The parameters have distinct roles:

- $\sigma_x \geq 0$: attenuation profile (zero at the physical/PML interface and increasing toward the outer boundary)
- $\kappa_x \geq 1$: stretching profile (affects matching and numerical behavior)
- $\alpha_x \geq 0$: frequency shift (improves low-frequency/DC behavior)

In the time domain, CPML can be written as a scaled derivative plus a memory term,

$$\frac{\partial u}{\partial \tilde{x}} = \frac{1}{\kappa_x}\frac{\partial u}{\partial x} + \psi_x, \tag{2}$$

where ψ_x is an auxiliary field updated in time (stored at Gauss points). FALCON uses this structure in both:

- the mechanical operator (displacement gradients / strain-displacement terms), and
- (for coupled / fully coupled analyses) the hydraulic operators used for pressure diffusion.

1.2 How the PML region is selected

PML is **geometric** and does not require element tags:

- FALCON computes the model **bounding box** from the initial (reference) node coordinates.
- For each Gauss point, it computes a **normalized depth** $s \in [0, 1]$ into the layer relative to the enabled outer faces.
- If $s > 0$, the Gauss point is inside the PML and receives nonzero CPML coefficients.

This means:

- You do not manually select elements.
- If you want PML outside your physical region, you must include elements there (the PML operates on existing elements).

1.3 Coefficient profiles and defaults

Across the layer, FALCON uses a polynomial profile in the normalized depth s:

$$p(s) = s^m, \qquad m \geq 1. \tag{3}$$

The Gauss-point coefficients then ramp as:

$$\sigma(s) = \sigma_{\max}\, p(s), \qquad \kappa(s) = 1 + (\kappa_{\max} - 1)\, p(s), \qquad \alpha(s) = \alpha_{\max}\, p(s). \tag{4}$$

This choice keeps the interface (where $s = 0$) close to the physical operator ($\sigma = 0$, $\kappa = 1$, $\alpha = 0$) and increases attenuation toward the outer boundary.

1.4 What PML changes in FALCON

1.4.1 Mechanics (displacement operator)

Inside the PML region:

- spatial derivatives are scaled by $1/\kappa$ (direction-wise), and
- memory variables ψ contribute additional history terms in the discrete weak form.

For dynamics, the PML attenuation profile also introduces an additional **velocity-proportional** contribution consistent with a $\sigma\, M$ term (assembled using Gauss-point σ).

1.4.2 Coupled / fully coupled (pressure diffusion)

For coupled and fully coupled analyses, the same CPML coefficients are applied to the pressure operators (PW, and PA in fully coupled). This means pressure transients are also attenuated inside the PML region using:

- the same $1/\kappa$ gradient scaling, and
- CPML history terms (memory variables) associated with pressure gradients.

1.5 Parameter roles and practical tuning

PML is a tuning problem: best values depend on wavelength content, mesh resolution, and how much buffer you can afford.

- **Thickness** L (`@@Thickness`)
  - Sets the physical width of the layer.
  - A thicker layer improves absorption of longer wavelengths and allows smoother ramps.
- **Attenuation magnitude** $\sigma_{\max}$ (`@@SigmaMax`)
  - Controls how strongly waves are damped near the outer boundary.
  - Too small $\rightarrow$ weak absorption; too large (especially with a thin layer) $\rightarrow$ stronger numerical reflection from impedance mismatch.
- **Stretching** $\kappa_{\max}$ (`@@KappaMax`, must be ≥ 1)
  - Modifies the effective derivative scaling through the layer.
  - Values slightly above 1 can help matching; very large values can degrade accuracy.
- **Frequency shift** $\alpha_{\max}$ (`@@AlphaMax`, must be ≥ 0)
  - Improves low-frequency absorption and stabilizes the $\omega \rightarrow 0$ behavior in (1).
  - Particularly important for step-like or very low-frequency loading.
- **Exponent** m (`@@Exponent`, must be ≥ 1)
  - Controls how quickly coefficients ramp. Larger m concentrates attenuation closer to the outer boundary.

1.6 Low-frequency / step-like loading (why drift can appear)

Two effects can combine in wave-type problems with “non-reflecting” boundaries:

1) **Physics (impedance) of a step traction**

A step traction contains a DC (zero-frequency) component. In a 1D acoustic/elastic impedance picture, a constant applied traction can correspond to a nearly constant particle velocity behind the wavefront,

$$\sigma_0 = Z\,v, \qquad Z = \rho c, \qquad v = \frac{\sigma_0}{\rho c}. \tag{5}$$

If that velocity persists, displacement grows approximately linearly:

$$u(t) = \int_0^t v\,dt = \frac{\sigma_0}{\rho c}\,t. \tag{6}$$

This behavior is not, by itself, a numerical instability; it is a consequence of sustained low-frequency content.

2) **CPML behavior as** $\omega \to 0$

From (1), the stretching function contains a term $\sigma/(\alpha + i\omega)$. If $\alpha = 0$, the response is poorly conditioned as $\omega \to 0$ for loads dominated by very low frequencies. Setting a **nonzero** $\alpha_{\max}$ makes the denominator finite in the DC limit, improving robustness for step-like/slow inputs.

Practical guidance:

- For absorption verification, prefer **transient, zero-mean** loading (short pulses / wavelets).
- If a step-like load is required, use a **nonzero** `@@AlphaMax` and validate that PML history variables remain bounded.

1.7 Input: `% PML Layer`

Place the PML section **after** `% Elements` (so Gauss points exist).

1.7.1 Section header

```
% PMLLayer
% PML Layer
% PML-Layer
```

The section ends at a line that is (after comment stripping and trimming) exactly `%%%` (for example, `%%%  # end` is accepted).

1.7.2 Directives

Inside the `% PML Layer` section:

- Only lines that start with `@` are parsed (other lines are ignored).
- Keys are case-insensitive.

- One or more leading `@` characters are accepted (e.g., `@Thickness:` and `@@Thickness:` are equivalent).
- A space after `:` is optional (e.g., `@@Thickness:2.0` and `@@Thickness: 2.0` are equivalent).

Directive	Required?	Default	Meaning
`@@Thickness`	Yes	—	Physical thickness `L` of the PML layer (measured inward from the enabled outer faces).
`@@SigmaMax`	Yes	—	Peak attenuation profile value at the outer boundary (σ`_max`).
`@@KappaMax`	Yes	— (must be `>= 1`)	Stretching factor maximum (κ`_max`).
`@@AlphaMax`	No	`0`	Frequency shift maximum (α`_max`, must be `>= 0`). Nonzero values help prevent low-frequency/DC drift for step-like loading.
`@@Exponent`	No	`2`	Profile exponent `m` (must be `>= 1`). Controls how quickly coefficients ramp across the layer.
`@@Sides`	No	`YMin`	Faces where PML is applied: one or more of `ALL`, `XMin`, `XMax`, `YMin`, `YMax`, `ZMin`, `ZMax`. For 2D/axisymmetric analyses, `ZMin`/`ZMax` are accepted but have no effect.

1.8 Notes on advanced use

- PML coefficients are assigned using the model bounding box in the original (reference) coordinates. In large-deformation workflows, the PML region does not automatically follow the updated geometry.
- On checkpoint restart, Gauss-point PML coefficients and memory variables are restored from the checkpoint. Changing `% PML Layer` in the input file will not change an in-progress restarted run unless the solver explicitly re-applies the PML assignment.