



AD FALCON API Manual

# Void-Ratio Multiplier( $k_e$ ) for Unsaturated Permeability

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## 1 Void-Ratio Multiplier ( $k_e$ ) for Unsaturated Permeability

In unsaturated flow models, it is often useful to separate:

- a saturation-dependent relative permeability  $k_r(S_e)$ , typically bounded by  $[0, 1]$ , and
- a void-ratio-dependent permeability multiplier  $k_e(e)$  that scales the hydraulic conductivity as the pore structure changes.

With anisotropic permeability, the effective permeability tensor components are applied in the form:

$$k_{\text{eff}} = k_{\text{aniso}} k_{\text{sat}} k_e(e) k_r(S_e) / \mu$$

where  $k_{\text{aniso}}$  is the directional multiplier from @AnisotropicPerm:,  $k_{\text{sat}}$  is the saturated permeability from @Perm:, and  $\mu$  is the phase viscosity.

### 1.1 The $k_e(e)$ multiplier

When enabled, the void-ratio multiplier is computed from the current void ratio  $e$  and a reference void ratio  $e_{\text{ref}}$ :

$$k_e(e) = \frac{e^3(1 + e_{\text{ref}})^2}{e_{\text{ref}}^3(1 + e)^2}$$

This is applied only when  $e_{\text{ref}} > 0$  and  $e > 0$ . Otherwise,  $k_e(e) = 1$  (no void-ratio scaling).

### 1.2 Available models

Two permeability models provide  $k_e(e) k_r(S_e)$  behavior:

- VanGenuchtenKe
- BrooksCoreyKe

Both models also support a lower bound  $k_{\text{min}}$  on the final multiplier to avoid numerically-zero conductivities:

$$\max\{k_e(e) k_r(S_e), k_{\text{min}}\}$$

### 1.3 Syntax

```
@Perm: VanGenuchtenKe m <m> k_sat <k_sat> [ke_ref <e_ref>] [k_min <k_min>]
@Perm: BrooksCoreyKe lambda <lambda> k_sat <k_sat> [ke_ref <e_ref>] [k_min
<k_min>]
```

Notes:

- $k_{e\_ref}$  and  $e\_ref$  are accepted as synonyms for the reference void ratio.
- If  $k_{e\_ref}/e\_ref$  is omitted (or set to 0), then  $k_e(e) = 1$  and the model reduces to the corresponding saturation-only  $k_r(S_e)$  form (with  $k_{min}$  still applied).

The multiplier  $k_e(e)$  has the classical Kozeny-Carman void-ratio form. In FALCON it is combined with either the Mualem-van Genuchten or Brooks-Corey/Burdine relative-permeability functions, depending on the selected @Perm: model.

## 1.4 References

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- Brooks, R. H., & Corey, A. T. (1964). *Hydraulic properties of porous media and their relation to drainage design*. Transactions of the ASAE, 7(1), 26-28.
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