AD FALCON API Manual

NorSand Model (saturated)

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Contents

1 NorSand Model (saturated)

NorSand is a **critical-state sand model** with an image pressure `Pim` controlling the yield surface size. The formulation documented here follows the critical-state interpretation summarized by Jefferies and Been (2016), with the state parameter as a governing variable for sand behaviour in the sense of Been, Jefferies, and Hachey (1991). This page documents the **input syntax**, the **mathematical formulation** used by the NorSand UMAT in this codebase, and the currently packaged `Norsand` FALCON mini.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example (log CSL):

```
@UMAT: path/to/NorSandModelUMAT.cpp path/to/NorSandModelUMAT.hpp Mechanical
\
    CSLForm=0 Gref=20000 Pref=100 nG=0.6 Nu=0.10 \
    Gamma=1.115 LambdaCSL=0.076 Patm=100 \
    Mtc=1.40 N=0.30 Xtc=2.50 H0=30 Hpsi=200 \
    R=1.0 S=1 P_min=1e-6 FTOL=1e-6 \
    CustomVariable=Pim
```

Example (power CSL):

```
@UMAT: path/to/NorSandModelUMAT.cpp path/to/NorSandModelUMAT.hpp Mechanical
\
    CSLForm=1 Gref=20000 Pref=100 nG=0.5 Nu=0.15 \
    Ca=0.90 Cb=0.14 Cc=0.15 Patm=100 \
    Mtc=1.28 N=0.30 Xtc=4.60 H0=100 Hpsi=625 \
    R=1.2 S=1 P_min=1e-6 FTOL=1e-6 \
    CustomVariable=Pim
```

For readability, examples are wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

1.2 Material parameters

Parameters are passed as `name=value` pairs in the `@UMAT:` line.

Table 1. Material parameters

Symbol	Keyword in input	Units	Required	Description
–	`CSLForm`	–	✓	CSL form selector (`0` log, `1` power).
G_{ref}	`Gref`	stress	✓	Shear modulus reference value.
P_{ref}	`Pref`	stress	✓	Reference pressure for $G(p)$.
n_G	`nG`	–	✓	Shear modulus exponent.
ν	`Nu`	–	✓	Poisson's ratio.
Γ	`Gamma`	–	✓*	Log CSL intercept (only if `CSLForm=0`).
λ	`LambdaCSL`	–	✓*	Log CSL slope (only if `CSLForm=0`).
C_a	`Ca`	–	✓*	Power CSL intercept (only if `CSLForm=1`).
C_b	`Cb`	–	✓*	Power CSL coefficient (only if `CSLForm=1`).
C_c	`Cc`	–	✓*	Power CSL exponent (only if `CSLForm=1`).
p_{atm}	`Patm`	stress	✓	Normalizing pressure for power CSL.
M_{tc}	`Mtc`	–	✓	Triaxial-compression CSL ratio.
N	`N`	–	✓	Operating ratio parameter.
X_{tc}	`Xtc`	–	✓	Dilation limit parameter.
H_0	`H0`	–	✓	Hardening parameter.
H_ψ	`Hpsi`	–	✓	Hardening parameter.

Symbol	Keyword in input	Units	Required	Description
R	`R`	–	✓	OCR helper for initialization (must satisfy `R>=1`).
–	`S`	–	✓	Softening flag (`0` or `1`).
P_{min}	`P_min`	stress	✓	Numerical lower bound for pressure-dependent terms.
–	`FTOL`	–	✓	Yield-surface tolerance.

* "Required" depends on `CSLForm`.

1.2.1 Optional integration controls

These keys are optional. If omitted, the UMAT defaults are used.

Keyword in input	Default	Description
`IntegrationScheme`	`0`	Exposed option for the standalone examples; `0` uses the explicit adaptive scheme.
`STOL`	`1e-4`	Local integration error tolerance for explicit adaptive substepping.
`DTMIN`	`1e-3`	Minimum adaptive substep size (in `(0,1]`).
`MaxSubsteps`	`200`	Maximum substeps for substepping schemes.
`MaxNewtonIter`	`200`	Maximum Newton iterations.
`ExternalCompressionSign`	`-1`	`-1` = compression negative in the host (FALCON default), `+1` = compression positive.

1.3 Custom state variables

Declare custom variables using `CustomVariable=` in the `@UMAT:` line.

Name	Required	Meaning
`Pim`	recommended	Image pressure controlling the yield surface size.

1.4 Stress invariants

Let $\boldsymbol{\sigma}$ be the effective stress tensor using FALCON's sign convention (tension positive). Define:

$$\sigma_m = \tfrac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma}), \qquad \boldsymbol{s} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}$$

$$J_2 = \tfrac{1}{2}\,\boldsymbol{s} : \boldsymbol{s}, \qquad J_3 = \det(\boldsymbol{s})$$

The deviatoric stress magnitude (UMAT output column `q`) is

$$q = \sqrt{3J_2} \geq 0$$

and the mean pressure used in all NorSand equations is

$$p = -\sigma_m$$

The Lode angle $\theta \in [-\pi/6, \pi/6]$ is computed from J_2, J_3 (standard definition; see MohrModel for the invariant conventions used across the manual). The work-conjugate invariant setting used by NorSand is consistent with the formulation discussed by Resende and Martin (1985).

1.5 Elastic law

NorSand uses pressure-dependent isotropic elasticity:

$$G(p) = G_{\mathrm{ref}} \left(\frac{\max(P_{\min}, p)}{P_{\mathrm{ref}}} \right)^{n_G}$$

$$K(p) = \frac{2(1+\nu)}{3(1-2\nu)}\, G(p)$$

The elastic stiffness matrix in Voigt form is the standard isotropic tensor:

$$\mathbf{D}^e = \begin{bmatrix} \lambda_e + 2\mu & \lambda_e & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e + 2\mu & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e & \lambda_e + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \lambda_e = K - \tfrac{2}{3}G, \ \mu = G$$

1.6 Critical state line (CSL) and state parameters

NorSand supports two CSL forms. Let e be the void ratio and define the CSL void ratio $e_c(p)$. This critical-state framing follows the standard critical-state soil mechanics basis of Schofield and Wroth (1968) and its NorSand interpretation in Jefferies and Been (2016).

1.6.1 Log CSL (`CSLForm=0`)

$$e_c(p) = \Gamma - \lambda \ln(\max(P_{\min}, p))$$

with λ from `LambdaCSL`.

1.6.2 Power CSL (`CSLForm=1`)

$$e_c(p) = C_a - C_b \left(\frac{\max(P_{\min}, p)}{p_{\text{atm}}} \right)^{C_c}$$

Define the (current) state parameter

$$\psi = e - e_c(p)$$

and the **image** state parameter (evaluated at `Pim`)

$$\psi_i = e - e_c(p_{im})$$

1.7 Lode-angle dependent critical friction ratio

The Lode-angle dependent critical friction ratio is

$$M(\theta) = M_{tc} \left(1 - \frac{M_{tc}}{3 + M_{tc}} \cos\left(\tfrac{3}{2}\theta + \tfrac{\pi}{4}\right) \right)$$

We denote $M_c = M(\theta)$.

1.8 Mapped dilation limit parameter

NorSand maps X_{tc} to a pressure-dependent limit parameter Ξ:

$$\Xi = \frac{X_{tc}}{1 - \lambda(p_{im})\, X_{tc}/M_{tc}}$$

where:

- for log CSL: $\lambda(p_{im}) = \lambda$ (constant),
- for power CSL:

$$\lambda(p_{im}) = -\frac{de_c}{d\ln p}\bigg|_{p=p_{im}} = C_b\, C_c \left(\frac{\max(P_{\min}, p_{im})}{p_{\text{atm}}}\right)^{C_c}$$

1.9 Operating friction ratio and dilatancy

Let

$$\eta = \frac{q}{p}$$

The operating friction ratio M_i depends on M_c, ψ_i, and Ξ. The state dependence of dilatancy and operating stress ratio follows the NorSand developments summarized by Jefferies and Been (2016), with the loose-side dilatancy discussion tied to Li and Dafalias (2000) and the general-stress-state form of the operating critical friction ratio tied to Jefferies and Shuttle (2011).

Dense side ($\psi_i < 0$):

$$M_i = M_c \left(1 + \frac{N\,\Xi}{M_{tc}}\psi_i\right)$$

Loose side ($\psi_i \geq 0$):

- default (extended Dafalias form):

$$M_i = M_c \left(1 - \frac{N\,\Xi}{M_{tc}}\psi_i\right)$$

- optional loose-side switch (Taylor–Bishop): if you input `Mtc < 0` in the UMAT parameters, the implementation uses $M_i = M_c$ for $\psi_i \geq 0$. This branch corresponds to the Taylor-Bishop style dissipation interpretation discussed by Bishop (1950) and later NorSand summaries.

The plastic dilatancy function is

$$D_p = M_i - \eta$$

1.10 Yield function

The NorSand yield function is (compression-positive $p > 0$ form):

$$f(\boldsymbol{\sigma}, p_{im}, e) = q - \eta_y \, p$$

with the yield stress ratio

$$\eta_y = M_i \left(1 + \ln\left(\frac{p_{im}}{p}\right)\right)$$

The stress state is:

- **elastic** if $f \leq \mathrm{FTOL}$
- **plastic** if $f > \mathrm{FTOL}$

1.11 Plastic potential and flow rule (stress–dilatancy form)

The UMAT uses a stress–dilatancy flow rule consistent with:

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = D_p$$

where:

- $\varepsilon_v^p = \varepsilon_{xx}^p + \varepsilon_{yy}^p + \varepsilon_{zz}^p$
- the implementation uses $\varepsilon_p^p = \varepsilon_v^p / 3$ as the p-conjugate strain invariant,
- and $d\varepsilon_q^p = d\lambda$ (because $\partial f / \partial q = 1$).

Accordingly, the plastic potential gradient is constructed so that:

$$\frac{\partial g}{\partial p} = \frac{D_p}{3}, \qquad \frac{\partial g}{\partial q} = 1$$

1.12 Hardening law for `Pim`

NorSand evolves the image pressure with plastic shearing. The use of `Pim` as an image stress and the associated hardening structure follow the NorSand presentation in Jefferies and Been (2016).

$$\dot{p}_{im} = B_{\text{iso}} \, \dot{\varepsilon}_q^p$$

The state-dependent hardening modulus is

$$H = H_0 - H_\psi \, \psi$$

and the hardening limit stress is

$$p_{mx} = p \, \exp\left(-\frac{X_{tc}}{M_{i,tc}}\psi\right)$$

where $M_{i,tc}$ is the operating friction ratio evaluated at triaxial compression ($\theta = \pi/6$).

The base hardening term is:

$$B_{\text{iso,base}} = H \left(\frac{p}{p_{im}}\right)\left(\frac{M_i}{M_{i,tc}}\right)(p_{mx} - p_{im})$$

1.12.1 Optional softening term (S)

If `S=1`, a softening term is applied for **undrained loading paths** when $D_p > 0$. In this implementation, the UMAT detects that branch from a near-zero imposed volumetric strain increment, rather than from a separate user-defined drainage flag:

$$S_{\text{soft}} = \omega \left(\frac{\eta}{M_i}\right)\left(\frac{K}{p}\right) D_p \, p_{im}$$

with

$$\omega = 1 - \lambda(p)\, X_{tc}/M_{tc}$$

and $\lambda(p)$ evaluated at the **current** pressure p (constant for log CSL; local slope for power CSL).

The final hardening term used by the UMAT is:

$$B_{\text{iso}} = B_{\text{iso,base}} - S_{\text{soft}}$$

1.12.2 Consistency hardening modulus

The scalar hardening modulus appearing in the consistency condition is:

$$K_p = -\frac{\partial f}{\partial p_{im}} \frac{d p_{im}}{d\lambda} = \left(M_i \frac{p}{p_{im}}\right) B_{\text{iso}}$$

1.13 Integration options

The standalone NorSand examples documented here use the explicit adaptive integration path. The exposed control is `IntegrationScheme = 0`, together with `STOL`, `DTMIN`, and `Max Substeps`.

1.14 Initialization and post-equilibrium conditioning (`Pim`)

After initialization (or after a geostatic/body-force equilibrium update), the stress state and void ratio may not be consistent with the currently stored `Pim`. The NorSand UMAT conditions `Pim` so the current stress state lies **on or inside** the yield surface.

- If `Pim` is missing, it is initialized from the current stress state and then enlarged using `R` (OCR helper).
- If `Pim` is present, it is only increased if needed (never shrunk).

This prevents starting a step from an overstressed state and avoids abrupt numerical corrections at the first constitutive update.

1.15 FALCON mini

The packaged mini tool id is `Norsand`. It lives under `mini_tools/Norsand`.

1.15.1 How to run

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool Norsand
--mini-input
/path/to/UMATLIB_FALCON/falcon_minis/Norsand/cases/TXD_dense_log_explicit
```

These examples are taken from the NorSand validation sets documented in the PLAXIS NorSand manual (Reference 7) and are provided here as packaged FALCON mini cases. The case families cover drained triaxial, undrained triaxial, and undrained direct simple shear (UDSS) loading paths, using the corresponding NorSand parameter sets for the logarithmic and power-form CSL examples.

Common material bundles used by the packaged examples:

Bundle	Used by	Constitutive parameters
Log CSL bundle	TXD/TXU log CSL, UDSS `S=0`, UDSS `S=1`	`CSLForm=0`, `Pref=100`, `Gref=20000`, `nG=0.6`, `Nu=0.1`, `Gamma=1.115`, `LambdaCSL=0.076`, `Patm=100`, `Mtc=1.40`, `N=0.30`, `Xtc=2.50`, `H0=30`, `Hpsi=200`, `R=1.0`

Bundle	Used by	Constitutive parameters
Power CSL bundle	TXU power CSL	`CSLForm=1`, `Pref=100`, `Gref=20000`, `nG=0.5`, `Nu=0.15`, `Ca=0.90`, `Cb=0.14`, `Cc=0.15`, `Patm=100`, `Mtc=1.28`, `N=0.30`, `Xtc=4.60`, `H0=100`, `Hpsi=625`, `R=1.2`

Bundled validation examples:

Validation family	Packaged cases	Purpose
TXD, log CSL	`cases/TXD_dense_log_explicit/input.txt`, `cases/TXD_loose_log_explicit/input.txt`	Dense and loose drained triaxial examples with logarithmic CSL.
TXU, log CSL	`cases/TXU_dense_log_explicit/input.txt`, `cases/TXU_loose_log_explicit/input.txt`	Dense and loose undrained triaxial examples with logarithmic CSL.
TXU, power CSL	`cases/TXU_dense_pow_explicit/input.txt`, `cases/TXU_loose_pow_explicit/input.txt`	Dense and loose undrained triaxial examples with power-form CSL.
UDSS, log CSL, `S=0`	`cases/UDSS_dense_log_S0_explicit/input.txt`, `cases/UDSS_loose_log_S0_explicit/input.txt`	Dense and loose UDSS examples with `S=0`.
UDSS, log CSL, `S=1`	`cases/UDSS_dense_log_S1_explicit/input.txt`, `cases/UDSS_loose_log_S1_explicit/input.txt`	Dense and loose UDSS examples with `S=1`.

1.15.2 Input syntax

`input.txt` uses whitespace-delimited `Key Value` pairs, for example:

```
Drainage Undrained
p0 250.0
psi0 -0.12
dEpsAxial -2e-4
```

```
nSteps 1000
CSLForm 0
```

The main driver selector is `Drainage`.

`Drainage` value	Meaning in the standalone mini
`Drained`	Axisymmetric drained triaxial loading with solved radial strain.
`Undrained`	Axisymmetric constant-volume triaxial loading, or UDSS-style loading if `dGammaXY != 0`.

Mini inputs used by the packaged cases:
Driver and loading controls:

Input key	Used by	Required / choices / defaults	Meaning
`CaseName`	all cases	Optional; label only	Driver case label.
`Drainage`	all cases	Required; choices `Drained` / `Undrained`	Selects drained or undrained loading.
`p0`	all cases	Required in packaged cases	Initial mean effective stress.
`psi0`	all cases	Required in packaged cases	Initial state parameter relative to the CSL.
`dEpsAxial`, `nSteps`	all cases	Required in packaged cases	Axial strain increment and number of loading steps.
`dGammaXY`	UDSS-style cases	Optional; `0.0` for triaxial examples	Engineering shear increment used by the UDSS examples.
`SchemeTag`	all packaged examples	Optional; packaged examples use `explicit`	Integration label used by the example inputs.
`InitializationMethodFlag`	all packaged examples	Optional; `1`, `2`, `3`	Per-step override for the UMAT integration scheme.

Input key	Used by	Required / choices / defaults	Meaning
`OutFile`	all cases	Optional; packaged examples use `stress_results.csv`	Output CSV name.

Initialization rules used by the packaged mini inputs:

- If `StressXX`, `StressYY`, or `StressZZ` is provided, the driver uses the direct-stress initialization path and reads `VoidRatio` directly.
- Otherwise, the driver initializes from `p0` and `psi0`, computes the CSL void ratio at `p0`, and sets `e_0 = e_c(p_0) + psi_0`.
- `psi0` is therefore an initialization input, not a history variable.
- `VoidRatio` is the actual state variable carried during the run.
- `Pim` is a custom variable because it is an internal history variable that must be stored and updated from step to step.

The same `Pim` initialization logic is part of the UMAT interface itself: if `Pim` is not supplied, the UMAT initializes it from the current stress state and void ratio before constitutive updates. In FEM workflows, this is the same mechanism used after initialization or after equilibrium/-gravity updates when the current stress state must remain admissible with respect to the stored image pressure.

NorSand constitutive parameters:

Input key	Used by	Required / choices / defaults	Meaning
`CSLForm`	all cases	Required; choices `0 = log`, `1 = power`	Selects logarithmic or power-form critical state line.
`Pref`, `Gref`, `nG`, `Nu`	all cases	Required in packaged cases	Elastic stiffness parameters.
`Gamma`, `LambdaCSL`	all cases	Required in packaged cases	Critical state line parameters.
`Ca`, `Cb`, `Cc`	all cases	Required in packaged cases	State-parameter and image-stress controls.

Input key	Used by	Required / choices / defaults	Meaning
`Patm`, `Mtc`, `N`, `Xtc`, `H0`, `Hpsi`, `R`, `S`	all cases	Required in packaged cases	Core NorSand constitutive parameters controlling stress ratio, image hardening, and dilatancy.
`P_min`, `FTOL`	all cases	Optional; driver safeguards if omitted	Numerical safeguards for low pressure and admissibility tolerance.

1.15.3 Hydromechanical assumptions

The packaged NorSand mini is an effective-stress mechanical driver:

- there is no intrinsic unsaturated constitutive coupling
- there is no suction or retention update
- drained and undrained differences come entirely from the loading constraint and the evolving NorSand state variables

So the mini is best read as a state-parameter sand model for triaxial effective-stress response, not as a hydraulic or unsaturated soil driver.

1.15.4 Sample input

Dense drained logarithmic-CSL example Path: `mini_tools/Norsand/cases/TXD_dense_log_explicit/input.txt`

```
CaseName TXD_dense_log
Drainage Drained
p0 250.0
psi0 -0.12
dEpsAxial -2e-4
nSteps 1000
OutFile stress_results.csv
CSLForm 0
Pref 100.0
Gref 20000.0
Nu 0.1
Gamma 1.115
LambdaCSL 0.076
Mtc 1.40
H0 30.0
```

```
Hpsi 200.0
```

This packaged case is the baseline dense drained NorSand example.

Companion validation inputs Representative companion cases:

- mini_tools/Norsand/cases/TXD_loose_log_explicit/input.txt
- mini_tools/Norsand/cases/TXU_dense_log_explicit/input.txt
- mini_tools/Norsand/cases/TXU_loose_log_explicit/input.txt
- mini_tools/Norsand/cases/TXU_dense_pow_explicit/input.txt
- mini_tools/Norsand/cases/TXU_loose_pow_explicit/input.txt
- mini_tools/Norsand/cases/UDSS_dense_log_S0_explicit/input.txt
- mini_tools/Norsand/cases/UDSS_dense_log_S1_explicit/input.txt
- mini_tools/Norsand/cases/UDSS_loose_log_S0_explicit/input.txt
- mini_tools/Norsand/cases/UDSS_loose_log_S1_explicit/input.txt

1.15.5 Output files and columns

Each packaged case writes `stress_results.csv`.

Output file	Produced by	Main use
`stress_results.csv`	all packaged runs	Main output written by the standalone mini.

Primary columns:

Output column	Meaning
`eps_xx`, `eps_yy`, `eps_zz`, `eps_xy`, `sxx`, `syy`, `szz`, `sxy`	Strain and stress components.
`p`, `q`	Mean and deviatoric stress invariants.
`voidRatio`	Void ratio.
`Pim`	Image stress / hardening variable.
`iplas`	Plastic loading indicator.
`PlasticStrainIncXX` to `PlasticStrainIncXY`	Cumulative plastic-strain components where the case writes them.

The following example inputs are also provided under `docs/Falcon_inputs/norsand/`:

Case	Explicit
TXU dense (log CSL)	TXU_dense_log_explicit.txt
TXU loose (log CSL)	TXU_loose_log_explicit.txt
TXD dense (log CSL)	TXD_dense_log_explicit.txt
TXD loose (log CSL)	TXD_loose_log_explicit.txt
UDSS dense (log CSL, S=0)	UDSS_dense_log_S0_explicit.txt
UDSS dense (log CSL, S=1)	UDSS_dense_log_S1_explicit.txt
UDSS loose (log CSL, S=0)	UDSS_loose_log_S0_explicit.txt
UDSS loose (log CSL, S=1)	UDSS_loose_log_S1_explicit.txt
TXU dense (power CSL)	TXU_dense_pow_explicit.txt
TXU loose (power CSL)	TXU_loose_pow_explicit.txt

The plots in the next section are generated directly from the bundled explicit NorSand mini cases and follow the same layout as the PLAXIS manual validation figures.

1.16 Results

The figures below are produced directly from the bundled explicit NorSand mini cases.

Dense and loose drained and undrained triaxial cases for the logarithmic CSL bundle (`p0 = 250 kPa`).

Dense and loose UDSS cases with `S=0` and `S=1` for the logarithmic CSL bundle (`p0 = 250 kPa`).

Dense and loose undrained triaxial cases for the power-form CSL bundle (`p0 = 200 kPa`).

1.17 Applications and limitations

- Best suited to state-parameter sand behavior in monotonic and cyclic effective-stress problems, especially dense-loose response comparisons.
- Appropriate for uncoupled and effective-stress-based coupled analyses when the hydraulic side is defined separately.
- It is not an intrinsic unsaturated constitutive law. The current packaged mini covers effective-stress triaxial and UDSS-style validation paths only.

1.18 References

1. Been, K., Jefferies, M. G., and Hachey, J. (1991). The critical state of sands. *Géotechnique*, 41(3), 365-381.

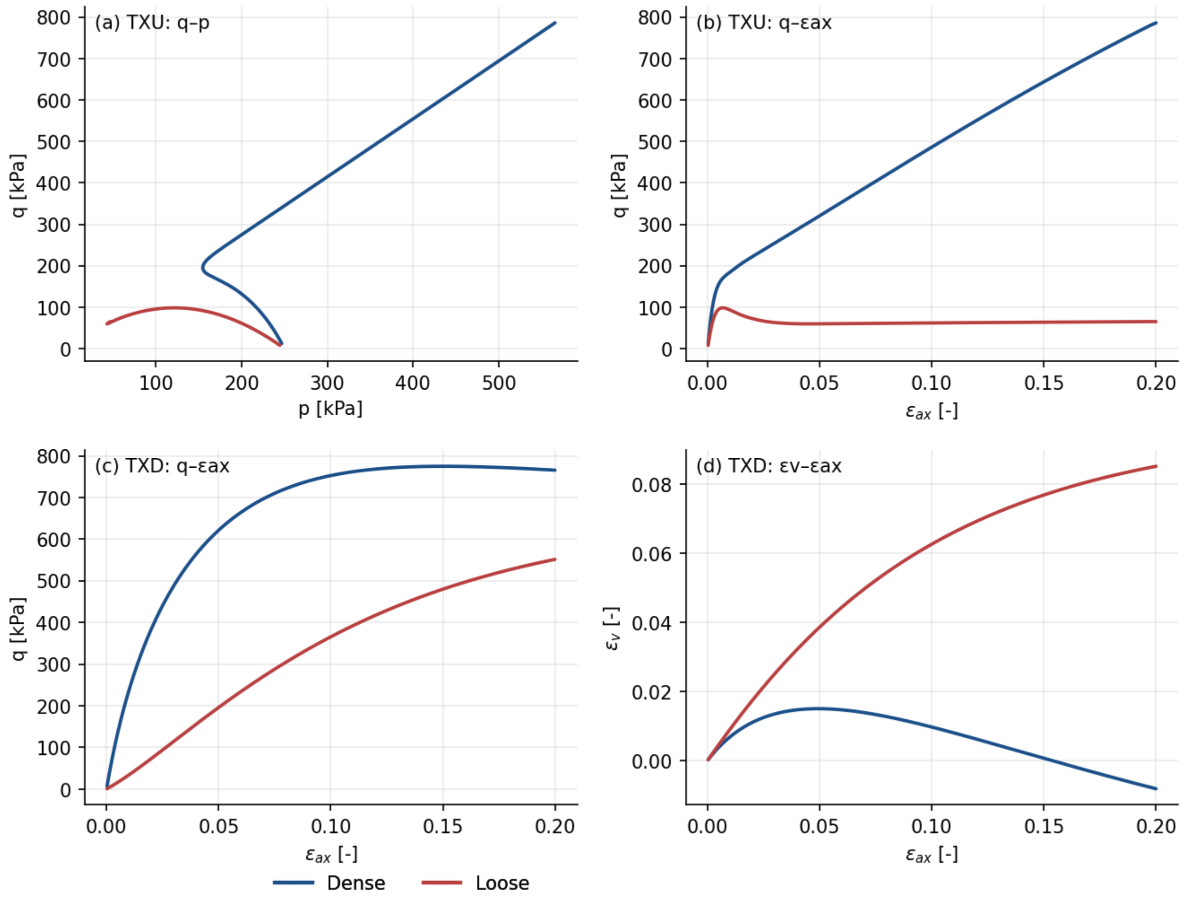

Figure 1: NorSand TXU/TXD results

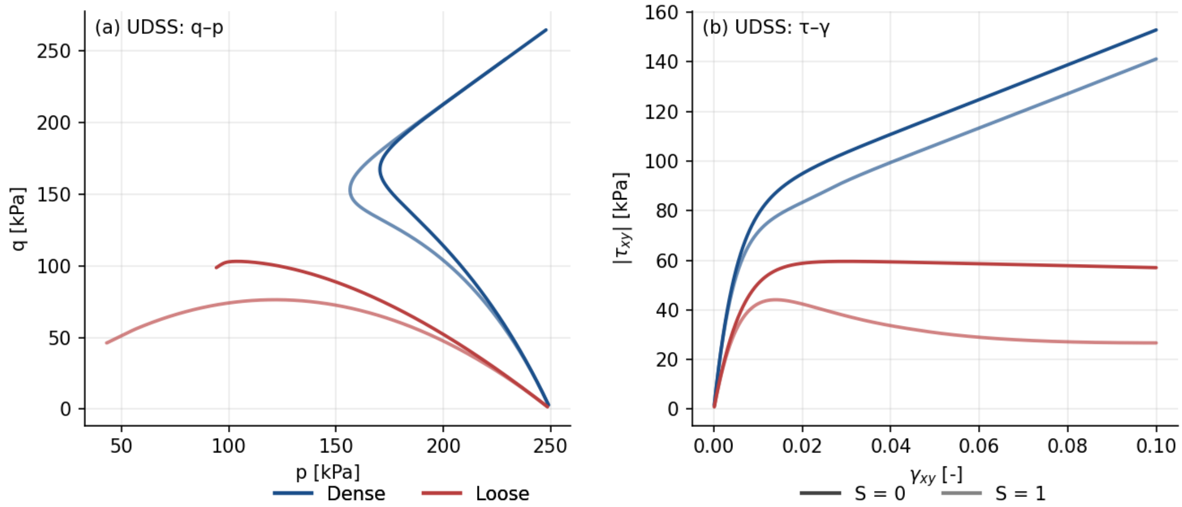

Figure 2: NorSand UDSS results

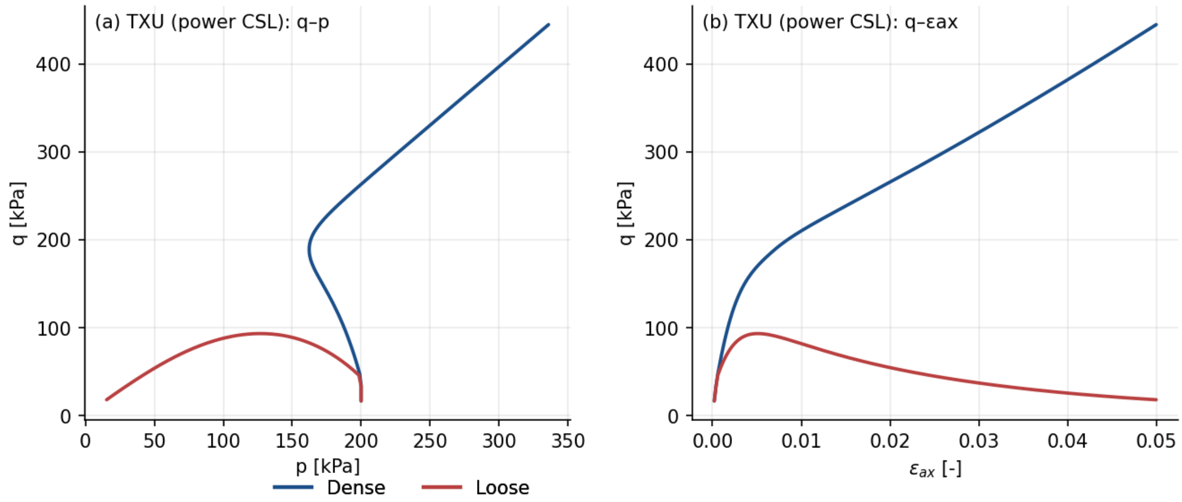

Figure 3: NorSand power CSL results

2. Bishop, A. W. (1950). Reply to discussion on measurement of shear strength of soils. *Géotechnique*, 2, 90-108.
3. Jefferies, M., and Been, K. (2016). *Soil Liquefaction: A Critical State Approach*. CRC Press.
4. Jefferies, M., and Shuttle, D. (2011). On the operating critical friction ratio in general stress states. *Géotechnique*, 61(8), 709-713.
5. Li, X. S., and Dafalias, Y. F. (2000). Dilatancy for cohesionless soils. *Géotechnique*, 50(4), 449-460.
6. Schofield, A. N., and Wroth, C. P. (1968). *Critical State Soil Mechanics*. McGraw-Hill.
7. Seequent. (2025). *PLAXIS 2025.1 User Defined Soil Models: NorSand — An elasto-plastic model for soil behaviour with static liquefaction*. Last updated September 24, 2025.