



AD FALCON API Manual

Mohr Hardening Model

Javad Ghorbani

March 26, 2026

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1 Mohr Hardening Model

The MohrHardeningUMAT extends the rounded Mohr-Coulomb model with strain-dependent hardening/softening of strength and an optional principal-stress tension cutoff. The model preserves the rounded Mohr shear surface and non-associated flow rule. This is a saturated-only formulation.

1.1 Syntax

This model is configured in % Materials as a user-defined mechanical material. Use @UMAT: with category Mechanical and pass the parameters as name=value pairs.

Example:

```
@UMAT: path/to/MohrHardeningUMAT.cpp path/to/MohrHardeningUMAT.hpp
Mechanical \
  E=180000 Nu=0.35 STOL=1e-3 FTOL=1e-6 LTOL=1e-4 \
  Phi_f=35 Cohesion=0 A=0.0038 Psi_f=33.5 \
  CapLambda=0.2 UseEllipticalCap=1 UseCompactionSwitch=0 Pc0=300
SmoothBeta=50 \
  Rf=0.10 TensionCutoff=-1 StrengthLaw=1 Phi0=5 Phi_res=30 Psi0=0
Psi_res=0 \
  Cohesion_res=0 EpsPq_peak=0.0038 EpsPq_soft=0.010 SoftReg=0 \
  CustomVariable=IsotropicHardening,EpsPq
```

For readability, this example is wrapped across multiple lines; in input files you should write the full @UMAT: directive on a single line.

Use the parameter names shown in the tables below.

1.2 Material parameters

Symbol	Keyword in input	Units	Required	Description
E	E	stress	✓	Young's modulus.
ν	Nu	–	✓	Poisson's ratio.
STOL	STOL	–	✓	Stress-integration tolerance.
FTOL	FTOL	–	✓	Yield-surface tolerance.

Symbol	Keyword in input	Units	Required	Description
LTOL	LTOL	–	✓	Load-unload de- tection tolerance.
φ_f	Phi_f	°	✓	Failure friction angle.
c	Cohesion	stress	✓	Cohesion inter- cept.
A	A	strain	✓	Hyperbolic hard- ening parameter (used by Strength Law = σ).
ψ_f	Psi_f	°	✓	Failure dilation angle.
λ_{cap}	CapLambda	–	✓	Reserved param- eter; accepted by the interface but does not control the active yield function.
–	UseEllipticalCap	–	✓	Reserved flag; ac- cepted by the in- terface but does not control the ac- tive yield function.
–	UseCompaction Switch	–	✓	Reserved flag; ac- cepted by the in- terface but does not control the ac- tive yield function.
p_{c0}	Pc0	stress	✓	Initial value assigned to Isotropic Hardening.
β	SmoothBeta	–	✓	Smoothing pa- rameter used only when blending the Mohr surface with an active tension cutoff.

Symbol	Keyword in input	Units	Required	Description
R_f	Rf	–	×	Hyperbolic rounding parameter for the rounded Mohr surface. Default 0.1.
σ_t	TensionCutoff	stress	×	Maximum admissible principal tensile stress. Set < 0 to disable. Default -1.
–	StrengthLaw	–	×	0: legacy hyperbolic hardening. 1: smooth hardening to peak followed by softening to residual.
φ_0	Phi0	°	×	Initial friction angle for StrengthLaw = 1.
φ_{res}	Phi_res	°	×	Residual friction angle for StrengthLaw = 1.
ψ_0	Psi0	°	×	Initial dilation angle for StrengthLaw = 1.
ψ_{res}	Psi_res	°	×	Residual dilation angle for StrengthLaw = 1.
c_{res}	Cohesion_res	stress	×	Residual cohesion for StrengthLaw = 1.
$\varepsilon_{q,peak}^p$	EpsPq_peak	strain	×	Plastic shear strain at peak strength for StrengthLaw = 1.

Symbol	Keyword in input	Units	Required	Description
$\Delta \varepsilon_{q,soft}^p$	EpsPq_soft	strain	×	Plastic shear strain interval over which residual strength is reached.
–	SoftReg	–	×	Optional non-negative regularization added to the consistent denominator in plastic correction.

1.2.1 Parent rounded-Mohr limit

The implementation reduces to the parent rounded Mohr response when:

- StrengthLaw = 0 and A = 0, or
- StrengthLaw = 1 with EpsPq_peak = 0 and EpsPq_soft = 0,
- and TensionCutoff < 0.

In that limit the active surface is the rounded Mohr surface only; the reserved cap inputs (CapLambda, UseEllipticalCap, UseCompactionSwitch) are accepted but inactive.

1.3 Custom state variables

This UMAT uses custom state variables to store the strain-history variables and optional diagnostics. Declare them using CustomVariable= in the @UMAT: line.

Required / primary history variables:

Name	Meaning
IsotropicHardening	Isotropic hardening variable, initialized from Pco.
EpsPq	Accumulated plastic shear strain driving the hardening / softening law.

Useful optional diagnostics:

- Phi_m
- Psi_m
- Cohesion_m
- YieldResidual

- Substeps
- SubstepFails
- NegDFailureFlag
- BracketingFailureFlag
- DriftFailureFlag

The standalone mini driver also reports several of these quantities directly in its CSV output.

1.4 Sign convention and stress measures

The code follows the usual FALCON tensor sign convention:

- stresses are tension-positive
- compressive stresses are therefore negative
- compressive strain increments are negative

The mean stress used internally is

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

so compression corresponds to negative σ_m . In triaxial interpretation you may prefer the compression-positive measure $p' = -\sigma_m$.

The deviatoric invariants are formed from

$$\mathbf{s} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}, \quad J_2 = \frac{1}{2} \mathbf{s} : \mathbf{s}, \quad J_3 = \det(\mathbf{s})$$

with

$$q = \sqrt{3J_2}$$

and the Lode angle extracted from J_2 and J_3 .

1.5 Rounded Mohr yield and flow

The active shear surface is the rounded Mohr-Coulomb surface already used in the parent model. In compact form,

$$F_s = \sigma_m \sin \phi_m + \sqrt{\bar{s}^2 K(\theta, \phi_m)^2 + (R_f \sin \phi_m)^2} - c_m \cos \phi_m$$

where:

- ϕ_m is the mobilized friction angle

- ψ_m is the mobilized dilation angle used in the plastic potential
- c_m is the mobilized cohesion
- R_f controls the hyperbolic rounding
- $K(\theta, \cdot)$ is the Abbo-Sloan style Lode-angle function used by the rounded Mohr implementation

The plastic potential uses the same rounded form, replacing ϕ_m with ψ_m .
This gives a non-associated flow rule except in the special case $\psi_m = \phi_m$.

1.6 Strength evolution law

The hardening variable is the accumulated plastic shear strain ε_q^p .

1.6.1 StrengthLaw = 0

The legacy hyperbolic law mobilizes friction according to

$$\tan \phi_m = \tan \phi_f \frac{\varepsilon_q^p}{\varepsilon_q^p + A}$$

and scales dilation proportionally:

$$\psi_m = \psi_f \frac{\phi_m}{\phi_f}$$

with constant cohesion.

1.6.2 StrengthLaw = 1

The current packaged examples use the smooth hardening-softening branch driven by the accumulated plastic shear strain

$$\varepsilon_q^p = \text{EpsPq}$$

Define

$$\varepsilon_p = \text{EpsPq_peak}, \quad \Delta\varepsilon_s = \text{EpsPq_soft}$$

and the smoothstep function

$$S(t) = \begin{cases} 0, & t \leq 0 \\ 3t^2 - 2t^3, & 0 < t < 1 \\ 1, & t \geq 1 \end{cases}$$

Then the mobilized friction angle is

$$\phi_m(\varepsilon_q^p) = \begin{cases} \phi_0 + (\phi_f - \phi_0) S\left(\frac{\varepsilon_q^p}{\varepsilon_p}\right), & 0 \leq \varepsilon_q^p \leq \varepsilon_p, \varepsilon_p > 0 \\ \phi_f + (\phi_{res} - \phi_f) S\left(\frac{\varepsilon_q^p - \varepsilon_p}{\Delta\varepsilon_s}\right), & \varepsilon_q^p > \varepsilon_p, \Delta\varepsilon_s > 0 \\ \phi_f, & \text{otherwise} \end{cases}$$

with

$$\phi_0 = \text{Phi}_0, \quad \phi_f = \text{Phi}_f, \quad \phi_{res} = \text{Phi}_{res}$$

The mobilized dilation angle follows the same structure:

$$\psi_m(\varepsilon_q^p) = \begin{cases} \psi_0 + (\psi_f - \psi_0) S\left(\frac{\varepsilon_q^p}{\varepsilon_p}\right), & 0 \leq \varepsilon_q^p \leq \varepsilon_p, \varepsilon_p > 0 \\ \psi_f + (\psi_{res} - \psi_f) S\left(\frac{\varepsilon_q^p - \varepsilon_p}{\Delta\varepsilon_s}\right), & \varepsilon_q^p > \varepsilon_p, \Delta\varepsilon_s > 0 \\ \psi_f, & \text{otherwise} \end{cases}$$

with

$$\psi_0 = \text{Psi}_0, \quad \psi_f = \text{Psi}_f, \quad \psi_{res} = \text{Psi}_{res}$$

The mobilized cohesion remains constant up to peak and then softens toward the residual value:

$$c_m(\varepsilon_q^p) = \begin{cases} c, & \varepsilon_q^p \leq \varepsilon_p \\ c + (c_{res} - c) S\left(\frac{\varepsilon_q^p - \varepsilon_p}{\Delta\varepsilon_s}\right), & \varepsilon_q^p > \varepsilon_p, \Delta\varepsilon_s > 0 \\ c, & \text{otherwise} \end{cases}$$

with

$$c = \text{Cohesion}, \quad c_{res} = \text{Cohesion}_{res}$$

So StrengthLaw = 1 gives:

- smooth hardening from the initial state to the peak state
- smooth softening from peak to residual over EpsPq_soft
- continuity of both the value and slope at the start and end of each transition because of the smoothstep interpolation

1.7 Optional tension cutoff

If `TensionCutoff < 0`, the active yield function is the rounded Mohr surface alone.

If `TensionCutoff >= 0`, the implementation computes the maximum principal stress σ_{max} and activates a tensile cutoff:

$$F_t = \sigma_{max} - \sigma_t$$

with $\sigma_t = \text{TensionCutoff}$. The total yield function is then a smooth log-sum-exp blend of the Mohr surface and the tension surface:

$$F = \frac{1}{\beta} \ln(\exp(\beta F_s) + \exp(\beta F_t))$$

where `SmoothBeta` is the blending parameter. This smoothing is only used when the tension cutoff is active.



1.8 Elastic law and return mapping

The elastic predictor uses linear isotropic elasticity:

$$K = \frac{E}{3(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)}$$

with the standard 6x6 isotropic stiffness in Voigt form.

The constitutive update uses:

1. elastic predictor
2. yield check on the rounded Mohr / tension surface
3. local plastic correction with substepping
4. a consistent elastoplastic tangent when the state is plastic

The internal history update is driven by:

- `EpsPq` through the mobilized strength law
- diagnostic counters such as `Substeps` and `SubstepFails`

`SoftReg` can be used to regularize the algorithmic denominator when the softening branch causes the consistent denominator to become too small.

1.9 FALCON mini

The packaged mini tool id is `MohrHardening`. It lives under `mini_tools/MohrHardening`.

1.9.1 How to run

Run any packaged analysis by passing its case directory:

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool
MohrHardening --mini-input
/path/to/UMATLIB_FALCON/falcon_minis/MohrHardening/cases/drained
```

Packaged simulation families:

- drained triaxial: [cases/drained/input.txt](#)
- undrained triaxial: [cases/undrained/input.txt](#)
- drained parameter examples: [cases/drained_phires25/input.txt](#), [cases/drained_soft004/input.txt](#), [cases/drained_psi10/input.txt](#)

1.9.2 Input syntax

`input.txt` uses whitespace-delimited Key Value pairs, one item per line, for example:

```
Mode Drained
E 180000
Phi_f 35
StrengthLaw 1
```

The main driver selector is Mode.

Mode value	Meaning in the standalone mini	Boundary-condition meaning
Drained	Triaxial loading with a radial strain solve at each increment.	The driver adjusts the radial strain so the confining stress stays close to the target radial stress.
Undrained	Saturated constant-volume triaxial loading.	The driver enforces $e_{xx} = e_{zz} = -0.5 e_{yy}$, so the total volumetric strain stays zero.

Mini inputs used by the packaged cases:

Input key	Used by	Required / choices / defaults	Meaning
Mode	all cases	Required; choices Drained and Undrained in the packaged mini	Selects the driver branch.
E, Nu	all cases	Required in packaged cases	Elastic constants for the predictor step.
STOL, FTOL, LTOL	all cases	Optional tolerances; packaged cases set them explicitly	Stress-integration, yield-surface, and load-detection tolerances.
Phi_f, Cohesion, A, Psi_f	all cases	Accepted by the interface; packaged cases set them explicitly for compatibility	Legacy peak-strength inputs used by the constitutive model.
Rf	all cases	Required in packaged cases	Rounded-Mohr hyperbolic smoothing parameter.
TensionCutoff	all cases	Optional; values < 0 disable cutoff; packaged baseline uses disabled cutoff	Principal-stress tensile limit; values < 0 disable the cutoff.
CapLambda, Use EllipticalCap, Use CompactionSwitch	all cases	Optional compatibility controls; switches typically use 0/1	Compatibility inputs accepted by the interface; retained for completeness even though the active packaged examples are controlled by the rounded-Mohr hardening branch.
Pco	all cases	Required in packaged cases	Initial value assigned to the Isotropic Hardening custom variable.

Input key	Used by	Required / choices / defaults	Meaning
SmoothBeta	all cases	Optional; relevant when tension cutoff is active	Smoothing parameter used when the tension cutoff branch is active.
StrengthLaw	all cases	Required; packaged examples use 1	Selects the strength-evolution branch. The packaged examples use Strength Law = 1.
Phi0, Phi_res	StrengthLaw = 1 cases	Required when StrengthLaw = 1	Initial and residual friction angles for the smooth hardening-softening branch.
Psi0, Psi_res	StrengthLaw = 1 cases	Required when StrengthLaw = 1	Initial and residual dilation angles for the smooth hardening-softening branch.
Cohesion_res	StrengthLaw = 1 cases	Required when StrengthLaw = 1	Residual cohesion in the softening branch.
EpsPq_peak	StrengthLaw = 1 cases	Required when StrengthLaw = 1	Plastic shear strain at peak strength.
EpsPq_soft	StrengthLaw = 1 cases	Required when StrengthLaw = 1	Plastic shear strain interval used to soften from peak to residual.
SoftReg	all cases	Optional; packaged cases set it explicitly	Optional regularization added to the consistent denominator in the softening branch.
StressXX, StressYY, StressZZ	all cases	Required in packaged cases	Initial total stress components. The packaged cases use an isotropic starting state.

Input key	Used by	Required / choices / defaults	Meaning
VoidRatio	all cases	Required in packaged cases	Initial void ratio tracked by the standalone driver.
dEpsAxial	all cases	Required in packaged cases	Axial strain increment applied at each load step.
nSteps	all cases	Required in packaged cases	Number of imposed load steps.
DriverSubsteps	mainly drained cases	Optional; packaged drained cases set it explicitly	Splits each imposed axial step into smaller boundary-condition solves. This is especially useful in post-peak softening.
BCMaxIt, BCRelTol, BCAbsTol	drained cases	Optional; meaningful only for Mode = Drained	Controls the radial-stress boundary-condition solve in the drained driver branch.

1.9.3 Hydromechanical assumptions

The packaged MohrHardening mini is purely saturated mechanical loading:

- no suction, retention, or effective-stress weighting is used in this mini
- Drained and Undrained refer only to the triaxial boundary condition applied by the standalone driver
- VoidRatio is updated and reported by the driver, but there is no unsaturated hydraulic coupling in this model

1.9.4 Sample input

Drained baseline example Path: mini_tools/MohrHardening/cases/drained/input.txt

```
Mode Drained
E 180000
Nu 0.35
STOL 0.001
```

```
FTOL 0.000001
LTOL 0.0001
Phi_f 35
Cohesion 0
Psi_f 33.5
Rf 0.10
TensionCutoff -1
CapLambda 0.2
UseEllipticalCap 1
UseCompactionSwitch 0
Pco 300
SmoothBeta 50
StrengthLaw 1
Phi0 5
Phi_res 30
Psi0 0
Psi_res 0
Cohesion_res 0
EpsPq_peak 0.0038
EpsPq_soft 0.010
SoftReg 0
StressXX -300
StressYY -300
StressZZ -300
VoidRatio 0.60
dEpsAxial -1e-4
nSteps 200
DriverSubsteps 10
BCMaxIt 60
BCRelTol 1e-10
BCAbsTol 1e-8
```

This is the baseline drained packaged case. It hardens from the initial friction state to peak, then softens smoothly to the residual state while the driver keeps the confining stress close to the target radial stress.

Undrained baseline example Path: [mini_tools/MohrHardening/cases/undrained/input.txt](#)

This packaged case uses the same strength parameters and initial stress state as the drained baseline, but switches the driver to the saturated constant-volume undrained branch.

Additional drained parameter examples used in the sensitivity plot:

- [mini_tools/MohrHardening/cases/drained_phires25/input.txt](#): lower residual friction angle

- `mini_tools/MohrHardening/cases/drained_soft004/input.txt`: shorter softening interval
- `mini_tools/MohrHardening/cases/drained_psi10/input.txt`: sustained positive dilation

1.9.5 Output files and columns

Each run writes `stress_results.csv`.

Output file	Produced by	Main use
<code>stress_results.csv</code>	all cases	Main step-by-step history used by the packaged Mohr Hardening figures.

Main columns in `stress_results.csv`:

Output column	Meaning
<code>step</code>	Load-step index written by the standalone driver.
<code>exx, eyy, ezz, ezy, ezx, exy</code>	Strain components for the accepted step state.
<code>sxx, syy, szz, szy, szx, sxy</code>	Stress components for the accepted step state.
<code>q, p</code>	Deviatoric and mean stress measures used by the triaxial plots.
<code>e</code>	Void ratio.
<code>pc</code>	Current isotropic hardening measure written by the driver.
<code>epsPq</code>	Cumulative plastic shear strain driving the strength evolution.
<code>phi_m, psi_m, coh_m</code>	Mobilized friction angle, dilation angle, and cohesion at the current step.
<code>F</code>	Yield-function value reported by the UMAT/-driver.
<code>sigma3_target</code>	Target radial stress used by the drained boundary-condition solve.
<code>sigma3_drift</code>	Residual drift of the radial stress from the target after the drained boundary-condition solve.

When reading the packaged outputs, the main columns to inspect are:

- q and p for the stress path
- epsPq, phi_m, psi_m, and coh_m for the hardening-softening law
- e for the volumetric response
- sigma3_drift if you want to check the quality of the drained boundary-condition solve

The plots in the next section are generated from these packaged case CSVs.

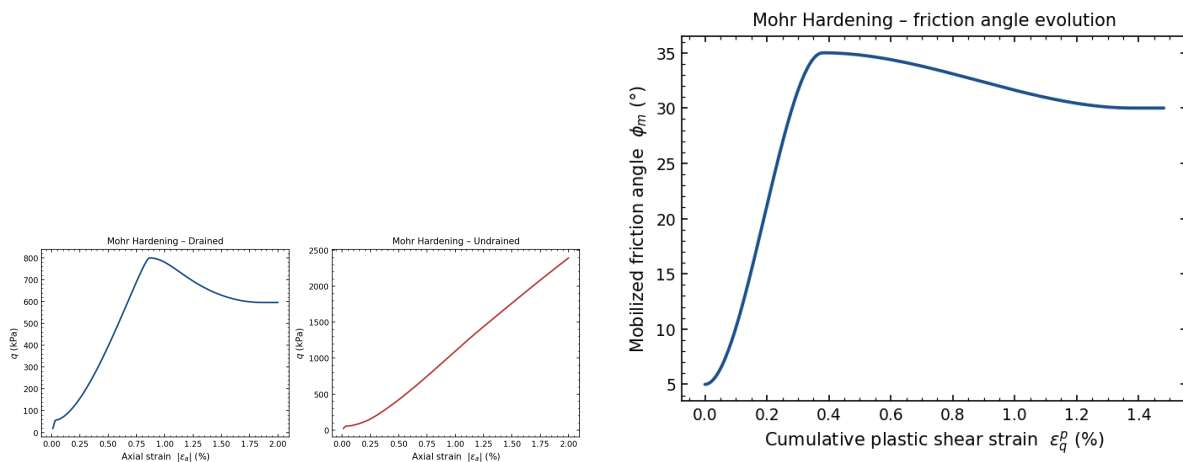
1.10 Results

The plots below are produced directly from the packaged MohrHardening case directories. The baseline cases use $E = 180 \text{ MPa}$, $\nu = 0.35$, $\phi_f = 35^\circ$, $\phi_{i0} = 5^\circ$, $\phi_{res} = 30^\circ$, $\psi_{i0} = \psi_{res} = 0^\circ$, $\epsilon_{pq_peak} = 0.0038$, and $\epsilon_{pq_soft} = 0.010$.

1.10.1 Baseline packaged cases

Bundled inputs:

- [mini_tools/MohrHardening/cases/drained/input.txt](#)
- [mini_tools/MohrHardening/cases/undrained/input.txt](#)



Left: deviatoric stress q versus axial strain for the packaged drained and undrained baseline cases. Right: mobilized friction angle phi_m versus cumulative plastic shear strain epsPq for the drained baseline case.

1.10.2 Drained parameter influence

Bundled inputs:

- [mini_tools/MohrHardening/cases/drained/input.txt](#)
- [mini_tools/MohrHardening/cases/drained_phires25/input.txt](#)

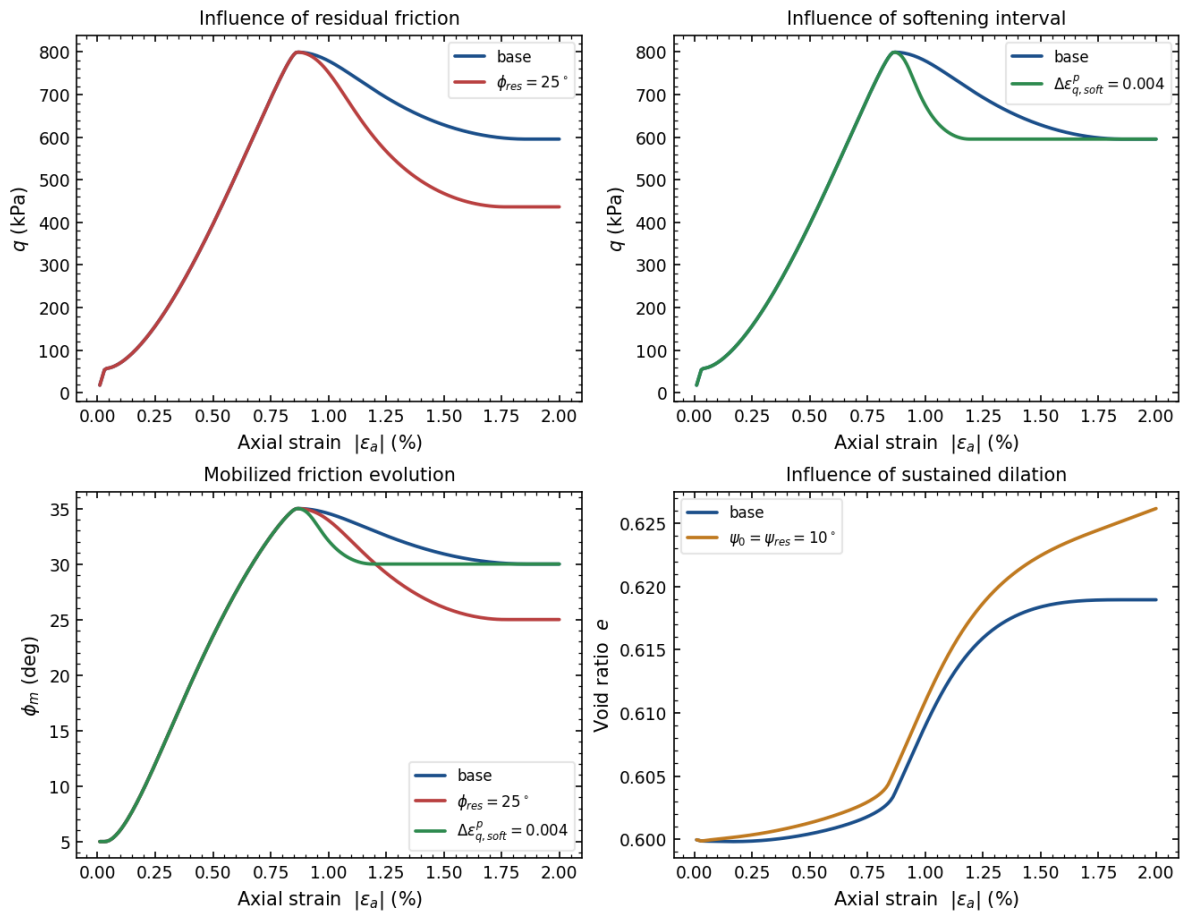


Figure 1: Drained MohrHardening parameter influence

- [mini_tools/MohrHardening/cases/drained_soft004/input.txt](#)
- [mini_tools/MohrHardening/cases/drained_psi10/input.txt](#)

Top left: lowering ϕ_{res} reduces the residual strength level reached after softening. Top right: reducing $\epsilon_{psq_soft}^p$ compresses the softening interval and makes the drop from peak to residual happen sooner. Bottom left: the corresponding mobilized friction-angle histories for the baseline, lower-residual, and faster-softening drained cases. Bottom right: sustained positive dilation ($\psi_0 = \psi_{res} = 10^\circ$) increases the void ratio compared with the baseline non-dilatant drained case.

1.11 Applications and limitations

- Best suited to frictional geomaterials that require peak, softening, and residual strength evolution without moving to a full critical-state framework.

- Useful for drained and undrained single-point response studies where the main requirement is strain-dependent friction, dilation, and cohesion degradation.
 - It is not an intrinsic unsaturated constitutive law and does not provide the fabric/state-variable structure of SANISAND-, NorSand-, or MIT-S1-type models.
-

1.12 References

- Abbo, A. and Sloan, S. (1995). *A smooth hyperbolic approximation to the Mohr-Coulomb yield criterion*.
- Sloan, S.W. (1987). *Substepping schemes for the numerical integration of elastoplastic stress-strain relations*.

