AD FALCON API Manual

MIT-S1 Model (Pestana & Whittle, 1999)

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Contents

1 MIT-S1 Model (Pestana & Whittle, 1999)

MIT-S1 is a unified effective-stress constitutive model for clays and sands, available through the `MITS1Model` UMAT. The implementation uses a critical-state, bounding-surface formulation with evolving anisotropy.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/MITS1Model.cpp path/to/MITS1Model.hpp Mechanical \
    Pa=100 rho_c=0.25 p_ref=100 theta=0.20 \
    Cb=750 K0NC=0.50 nu0=0.25 omega=1.25 omega_s=4.0 \
    phi_cs=32 phi_mr=30 p=2.0 m=0.8 psi=30 \
    D=0 r=1 h=0 enableOCMapping=1 \
    P_min=1e-9 FTOL=1e-4 stressRelTol=1e-5 maxSubsteps=500 \
    CustomVariable=Alpha,b_xx,b_yy,b_zz,b_zy,b_zx,b_xy,alpha0_star,␣
alpha0_i,p_srp,srp_has_reversal,eta_srp_xx,eta_srp_yy,eta_srp_zz,␣
eta_srp_zy,eta_srp_zx,eta_srp_xy,eps_srp_xx,eps_srp_yy,eps_srp_zz,␣
eps_srp_zy,eps_srp_zx,eps_srp_xy
```

For readability, this example is wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

Example inputs used on this page:

- Generic `% Materials` snippet: `materials_mits1.txt`
- Pestana & Whittle (1999) Fig. 13-style \(K_0\) undrained clay case: `pestana1999_fig13_ck0_strength_matching.txt`
- Pestana & Whittle (1999) Fig. 15-style \(K_0\) undrained clay case: `pestana1999_fig15_ck0_psi30.txt`

1.2 Material parameters

Table 1. Core MIT-S1 material parameters

Symbol	Keyword in input	Required	Default	Role in the model
p_a	Pa	✓	none	Reference pressure used to normalise stiffness and stress levels in Eqs. (1), (15), and (19).
ρ_c	`rho_c`	✓	none	LCC compressibility; controls the slope of the limiting compression curve in Eq. (2) and enters isotropic hardening and flow through Eqs. (9), (13), and (25b).
p'_{ref}	`p_ref`	✓	none	Fixes the LCC position in Eq. (2), so it sets where isotropic compression and δ_b calculations are anchored.
θ	`theta`	✓	none	Transition exponent governing how quickly the response approaches the LCC in Eqs. (1), (8b), (9), and (25b).
C_b	Cb	✓	none	Elastic stiffness parameter controlling $K_{\max}$ and ρ_s in Eqs. (15) and (19).

Symbol	Keyword in input	Required	Default	Role in the model
K_0^{NC}	`K0NC`	✓	none	Defines the normally consolidated stress ratio and enters the anisotropy/flow scalars in Eqs. (8c) and (13).
ν_0	`nu0`	✓	none	Poisson ratio at stress reversal; sets the elastic ratio $2G_{\max}/K_{\max}$ in Eq. (15b) and the evolving ν in Eq. (18).
ω	`omega`	✓	none	Controls shear/Poisson degradation away from the SRP through Eqs. (16), (18), and (20).
ω_s	`omega_s`	✓	none	Controls small-strain shear degradation and the swelling slope in Eqs. (19b) and (20).
φ'_{cs}	`phi_cs`	✓	none	Critical-state friction angle; defines the failure surface in Eq. (3), the LCC coupling constant α_0^2 in Eq. (10), and the reference branch in Eq. (5).

Symbol	Keyword in input	Required	Default	Role in the model
φ'_{mr}	`phi_mr`	✓	none	Reference peak friction angle at $e = 1$; controls the aperture scalar through Eqs. (4c) and (5).
n_p	`p`	✓	none	Void-ratio exponent in Eq. (5); governs how strongly density changes the peak friction angle.
m	`m`	✓	none	Bounding-surface slenderness parameter; appears directly in the surface shape, flow, and mapping laws in Eqs. (4a), (7), (12a), and (24a).
ψ	`psi`	✓	none	Kinematic hardening rate of the anisotropy tensor b in Eq. (7); it controls how fast the surface translates.
D	`D`	×	`0`	Hysteretic swelling contribution after stress reversal in Eq. (19b).
r	`r`	×	`1`	Exponent controlling the decay of the hysteretic term $D(1 - \mu^r)$ in Eq. (19b).

Symbol	Keyword in input	Required	Default	Role in the model
h	h	×	0	Overconsolidated mapping modulus scale in Eq. (25b).

Table 2. Numerical and advanced controls

Keyword in input	Required	Default	Role
`P_min`	✓	none	Positive floor for p' and α' to avoid singular stress-ratio updates.
`FTOL`	✓	none	Admissibility tolerance for the yield function and local correction steps.
`stressRelTol`	×	`1e-5`	Relative stress target used by the adaptive substepping error estimator.
`maxSubsteps`	×	`500`	Maximum number of internal substeps per increment.

Keyword in input	Required	Default	Role
`enableOCMapping`	×	`1`	Switches the inside-surface overconsolidated mapping on or off. When `1`, a trial state that remains inside the bounding surface can still generate plastic response by mapping the current stress point to a homothetic image point on the loading surface at fixed stress-ratio direction, anisotropy tensor, and void ratio, then evaluating the plastic modulus from that mapped state. When `0`, inside-surface increments are treated as purely elastic until the stress path reaches the bounding surface itself.
`Alpha_max`	×	`1e12`	Safety cap on α' during numerical integration.
`hardeningRateCap`	×	`50`	Caps logarithmic hardening rates as a numerical safeguard.
`alphaOverPMax`	×	`0`	Optional cap on α'/p'; `0` disables the cap.
`alphaCapScale`	×	`0`	Optional LCC-based cap on α' growth; `0` disables it.
`initOverride`	×	`0`	Rebuilds internal state from the current stress/void-ratio state during conditioning.

Keyword in input	Required	Default	Role
`initOCR`	×	1	OCR-style initialization multiplier used when `initOverride=1`. With `initAlphaMode=1`, the implementation sets $\alpha' = \texttt{initOCR}\, p'$, so the OCR-like ratio α'/p' is prescribed directly. With `initAlphaMode=0`, it sets $\alpha' = \texttt{initOCR}\, \alpha'_{\mathrm{ref}}$ with α'_{ref} taken from the current LCC-based reference state.
`initBScale`	×	1	Scale factor for rebuilding b from the current stress ratio.
`initAlphaMode`	×	1	Chooses how conditioning reconstructs α' from the current state.
`initEnforceAdmissible`	×	1	Enlarges α' during conditioning if the current state is inadmissible.

1.3 Custom state variables

Declare custom state variables using `CustomVariable=` so they are stored for restart/output.

Table 3. MIT-S1 custom variables

Keyword(s)	State variable	Required	Role
`Alpha`	α'	✓	Current bounding-surface size used in Eqs. (4a), (6b), (6c), (8b), and (12), and updated by Eqs. (9) to (11).

Keyword(s)	State variable	Required	Role
b_xx, b_yy, b_zz, b_zy, b_zx, b_xy	$\boldsymbol{b}$	✓	Components of the anisotropy tensor that translate the surface in Eq. (4a) and evolve through Eq. (7).
alpha0_star	α_0^*	✓	Current constant-η image-point size from Eq. (24a), used in the mapping factor g_1 in Eq. (24).
alpha0_i	α_{0i}	✓	First-yield loading-surface size used with α_0^* in Eq. (24) during overconsolidated mapping.
p_srp	p'_{srp}	✓	Mean effective stress at the stored stress-reversal point, used in Eqs. (16), (17), and (19b).
srp_has_reversal	SRP flag	✓	Indicates whether the stress-reversal-point history has been initialized for the hysteretic elasticity update.
eta_srp_xx, eta_srp_yy, eta_srp_zz, eta_srp_zy, eta_srp_zx, eta_srp_xy	$\boldsymbol{\eta}_{\text{srp}}$	✓	Stress-ratio tensor at the stress-reversal point, used to evaluate m_4 in Eq. (17).
eps_srp_xx, eps_srp_yy, eps_srp_zz, eps_srp_zy, eps_srp_zx, eps_srp_xy	$\boldsymbol{\varepsilon}_{\text{srp}}$	✓	Strain tensor at the stress-reversal point, used by the loading-unloading detector in Eq. (21).

1.4 Stress invariants and notation

Stress invariants are written in effective-stress form:

$$p' = \tfrac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma}'), \qquad \boldsymbol{s} = \boldsymbol{\sigma}' - p'\boldsymbol{I}, \qquad \boldsymbol{\eta} = \frac{\boldsymbol{s}}{p'}. \tag{0}$$

The third invariant of the stress-ratio tensor is

$$J_3^{\eta} = \det(\boldsymbol{\eta}) = \frac{\det(\boldsymbol{s})}{(p')^3}. \tag{0a}$$

The internal variables are the void ratio e, the scalar bounding-surface size α', and the deviatoric anisotropy tensor $\boldsymbol{b}$.

Plastic flow is written as

$$d\boldsymbol{\varepsilon}^p = d\lambda\,\boldsymbol{P}, \qquad d\varepsilon_v^p = d\lambda\,P_p, \qquad d\boldsymbol{\varepsilon}_d^p = d\lambda\,\boldsymbol{P}_s. \tag{0b}$$

1.5 LCC Compression Model (Eqs. 1–2)

For hydrostatic first loading:

$$d\varepsilon_v = \frac{e}{1+e}\left[\frac{\delta^\theta}{C_b\left(\dfrac{p'}{p_a}\right)^{1/3}} + \frac{\rho_c}{\left(\dfrac{p'}{p_a}\right)}\left(1-\delta^\theta\right)\right]\frac{dp'}{p_a}, \tag{1a}$$

$$d\varepsilon_v^e = \frac{e}{1+e}\,\frac{1}{C_b\left(\dfrac{p'}{p_a}\right)^{1/3}}\,\frac{dp'}{p_a}. \tag{1b}$$

The LCC distance measure is

$$\delta = 1 - \frac{p'}{p'_b}, \qquad p'_b = p'_{\mathrm{ref}}\left(\frac{1}{e}\right)^{1/\rho_c}. \tag{2}$$

1.6 Critical-State Failure and Bounding Surface (Eqs. 3–5)

The Matsuoka-Nakai critical-state failure function is

$$h_f(\boldsymbol{\eta}) = k^2 - \boldsymbol{\eta} : \boldsymbol{\eta} = 0, \tag{3a}$$

with

$$k^2 = k_a^2 + \left(3 - \frac{k_a^2}{2}\right) J_3^\eta, \qquad k_a^2 = \frac{8\sin^2\varphi'_{cs}}{3+\sin^2\varphi'_{cs}}. \tag{3b}$$

The distorted lemniscate bounding surface is

$$f = (p')^2 \left[(\boldsymbol{\eta} - \boldsymbol{b}) : (\boldsymbol{\eta} - \boldsymbol{b}) - \zeta^2 \left(1 - \left(\frac{p'}{\alpha'}\right)^m\right)\right] = 0, \tag{4a}$$

$$\zeta^2 = c^2 + \boldsymbol{b} : \boldsymbol{b} - 2\,\boldsymbol{\eta} : \boldsymbol{b}, \tag{4b}$$

$$c^2 = c_a^2 + \left(3 - \frac{c_a^2}{2}\right) J_3^\eta, \qquad c_a^2 = \frac{8\sin^2\varphi'_m}{3+\sin^2\varphi'_m}. \tag{4c}$$

The density-dependent peak friction angle is

$$\frac{1}{\tan\varphi'_m} = \frac{1}{\tan\left(45^\circ + \dfrac{\varphi'_{cs}}{2}\right)} + \left[\frac{1}{\tan\varphi'_{mr}} - \frac{1}{\tan\left(45^\circ + \dfrac{\varphi'_{cs}}{2}\right)}\right] e^{n_p}. \tag{5}$$

Here e^{n_p} means void ratio raised to the power n_p, not an exponential.

1.7 Yield Gradient and Hardening (Eqs. 6–11)

Define

$$\boldsymbol{Q} = \frac{\partial f}{\partial \boldsymbol{\sigma}'} \equiv (Q_p, \boldsymbol{Q}_s). \tag{6a}$$

With $g = (p'/\alpha')^m$, the components used in the UMAT are

$$Q_p = p' \left[(m\zeta^2 + 2\,\boldsymbol{\eta} : \boldsymbol{b})g - 2\,\boldsymbol{\eta} : \boldsymbol{\eta} + \left(9 - \frac{3c_a^2}{2}\right)(1-g)J_3^\eta\right], \tag{6b}$$

$$\boldsymbol{Q}_s = p' \left[2(\boldsymbol{\eta} - g\boldsymbol{b}) - \left(3 - \frac{c_a^2}{2}\right)(1-g)\frac{\partial J_3^\eta}{\partial \boldsymbol{\eta}}\right]. \tag{6c}$$

Kinematic hardening of the anisotropy tensor follows

$$d\boldsymbol{b} = \psi\,\frac{1+e}{e\,\alpha'}\left[\frac{r_x}{m}\langle Q_p\, d\varepsilon_v^p\rangle + r_y\left|\boldsymbol{Q}_s : d\varepsilon_d^p\right|\right](\boldsymbol{\eta} - \boldsymbol{b}), \tag{7}$$

with Macaulay brackets $\langle x\rangle = \max(x, 0)$ and

$$r_x = \frac{k^2 + \boldsymbol{b} : \boldsymbol{b} - 2\,\boldsymbol{\eta} : \boldsymbol{b}}{k_a^2}, \tag{8a}$$

$$r_y = \left(d^2 + \boldsymbol{b} : \boldsymbol{b} - 2\,\boldsymbol{\eta} : \boldsymbol{b}\right) \left[1 + \left(\frac{\alpha'}{p'} - 1\right) \delta_b^{\theta}\right], \tag{8b}$$

$$d^2 = d_a^2 + \left(3 - \frac{d_a^2}{2}\right) J_3^{\eta}, \qquad d_a^2 = \frac{2(1 - K_0^{NC})^2}{1 + K_0^{NC} + (K_0^{NC})^2}. \tag{8c}$$

The isotropic hardening law for α' is

$$\frac{d\alpha'}{\alpha'} = \frac{1 + e}{e(\rho_c - \rho_s)(1 - \delta_b)^{\theta}} \left[d\varepsilon_v^p + \delta_b^{\theta} \left(\frac{\boldsymbol{Q}_s : d\varepsilon_d^p}{p'}\right)\right] - \frac{2\,\boldsymbol{b} : d\boldsymbol{b}}{\alpha_0^2 + \boldsymbol{b} : \boldsymbol{b}}, \tag{9}$$

where

$$\delta_b = 1 - \frac{\alpha'}{p_b'} \left(1 + \frac{\boldsymbol{b} : \boldsymbol{b}}{\alpha_0^2}\right), \qquad \alpha_0^2 = 24 \left(\frac{\sin \varphi_{cs}'}{3 - \sin \varphi_{cs}'}\right)^2. \tag{10}$$

In the LCC regime ($\delta_b = 0$),

$$\frac{d\alpha'}{\alpha'} = \frac{1 + e}{e(\rho_c - \rho_s)}\, d\varepsilon_v^p - \frac{2\,\boldsymbol{b} : d\boldsymbol{b}}{\alpha_0^2 + \boldsymbol{b} : \boldsymbol{b}}. \tag{11}$$

1.8 Non-Associated Flow and Consistency (Eqs. 12–14)

The volumetric flow component is

$$P_p = \begin{cases} (k^2 - \boldsymbol{\eta} : \boldsymbol{\eta}) \dfrac{p'}{\alpha'} (1 - \delta_b)^m, & \boldsymbol{\eta} : \boldsymbol{\eta} \le k^2, \\ (k^2 - \boldsymbol{\eta} : \boldsymbol{\eta}) \dfrac{p'}{\alpha'}, & \boldsymbol{\eta} : \boldsymbol{\eta} > k^2, \end{cases} \tag{12a}$$

and the deviatoric flow component is

$$\boldsymbol{P}_s = x\, P_p\, \boldsymbol{\eta} + \frac{\zeta^2 \|\boldsymbol{\eta}\|}{\alpha'} \boldsymbol{Q}_s. \tag{12b}$$

The coefficient imposing the K_0^{NC} constraint is

$$x = \left(\frac{\rho_c}{\rho_c - \rho_s}\right) \left[\frac{1 + 2K_0^{NC}}{3(1 - K_0^{NC})} - \frac{K}{2G} \frac{\rho_s}{\rho_c}\right]. \tag{13}$$

Consistency gives

$$d\lambda\, H = -\frac{\partial f}{\partial \alpha'}\, d\alpha' - \frac{\partial f}{\partial \boldsymbol{b}} : d\boldsymbol{b}, \tag{14a}$$

$$d\lambda = \frac{K Q_p\, d\varepsilon_v + 2G\, \boldsymbol{Q}_s : d\varepsilon_d + \dfrac{\partial f}{\partial e}(1 + e)\, d\varepsilon_v}{H + K Q_p P_p + 2G\, \boldsymbol{Q}_s : \boldsymbol{P}_s}. \tag{14b}$$

1.9 Hysteretic Elasticity (Eqs. 15–21)

At the stress-reversal point,

$$\frac{K_{\max}}{p_a} = C_b \left(\frac{1+e}{e}\right) \left(\frac{p'}{p_a}\right)^{1/3} \left(1 + \frac{K_{\max}}{2G_{\max}} \boldsymbol{\eta} : \boldsymbol{\eta}\right)^{1/6}, \tag{15a}$$

$$\frac{2G_{\max}}{K_{\max}} = 3\left(\frac{1-2\nu_0}{1+\nu_0}\right). \tag{15b}$$

The shear/bulk ratio degrades with distance from the SRP:

$$\frac{(2G/K)}{(2G_{\max}/K_{\max})} = \begin{cases} \dfrac{1}{1+\omega m_4}, & p' < p'_{\mathrm{srp}}, \\ \dfrac{1}{1+\omega\mu m_4}, & p' \geq p'_{\mathrm{srp}}, \end{cases} \tag{16}$$

with

$$\mu = \begin{cases} p'/p'_{\mathrm{srp}}, & p' < p'_{\mathrm{srp}}, \\ p'_{\mathrm{srp}}/p', & p' \geq p'_{\mathrm{srp}}, \end{cases} \qquad m_4 = \left[(\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{srp}}) : (\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{srp}})\right]^{1/2}. \tag{17}$$

The equivalent Poisson ratio is

$$\nu = \nu_0 + \frac{\frac{1}{3}\omega m_4 (1+\nu_0)}{1 + \frac{2}{3}\omega m_4 (1+\nu_0)}, \qquad \nu_0 \leq \nu < 0.5. \tag{18}$$

The tangent bulk modulus and swelling slope are

$$\frac{K}{p_a} = \frac{1+e}{e\,\rho_s}\frac{p'}{p_a}, \tag{19a}$$

$$\rho_s = D(1-\mu^r) + \frac{1+\omega_s m_4}{C_b}\left(1 + \frac{K_{\max}}{2G_{\max}} \boldsymbol{\eta} : \boldsymbol{\eta}\right)^{1/6} \left(\frac{p'}{p_a}\right)^{2/3}. \tag{19b}$$

and the small-strain shear reduction is

$$\frac{G}{G_{\max}} = \frac{1}{(1+\omega m_4)(1+\omega_s m_4)}. \tag{20}$$

The SRP is identified from the strain increment relative to the previous reversal state:

$$s\,ds = \begin{cases} \Delta\varepsilon_v\, d\varepsilon_v, & d\varepsilon_v \neq 0, \\ \Delta\boldsymbol{\varepsilon}_d : d\boldsymbol{\varepsilon}_d, & d\varepsilon_v = 0, \end{cases} \qquad \begin{cases} ds > 0 & \text{loading}, \\ ds < 0 & \text{unloading (new SRP)}. \end{cases} \tag{21}$$

1.10 Bounding-Surface Mapping for Overconsolidated States (Eqs. 22–25)

Loading is detected from the image-point gradient:

$$K Q_p^I \, d\varepsilon_v + 2G \, \boldsymbol{Q}_s^I : d\boldsymbol{\varepsilon}_d \begin{cases} \geq 0 & \text{loading,} \\ < 0 & \text{unloading.} \end{cases} \tag{22}$$

When mapping is active,

$$\boldsymbol{P} = (1 - g_1)\boldsymbol{P}^I + g_1 \boldsymbol{P}_0, \tag{23a}$$

$$H = \langle H^I \rangle + H_0 \frac{g_1}{1 - g_1} \left(1 - \frac{\boldsymbol{\eta} : \boldsymbol{\eta}}{c^2}\right)^{1/2}, \tag{23b}$$

$$g_1 = \frac{\alpha' - \alpha_0^*}{\alpha' - \alpha_{0i}}, \qquad 0 \leq g_1 \leq 1, \tag{24}$$

with first-yield quantities

$$P_{0p} = -2\|\boldsymbol{\eta} - \boldsymbol{b}\| \left\| \frac{\boldsymbol{s}^I}{\alpha'} \right\|, \qquad \boldsymbol{P}_{0s} = \boldsymbol{P}_s^I, \tag{25a}$$

$$H_0 = \left(\frac{\rho_s^I}{\rho_c - \rho_s^I} \right) \frac{h}{1 - \delta_b^\theta} K_{\max}^I \|\boldsymbol{Q}^I\| \|\boldsymbol{P}^I\|. \tag{25b}$$

The loading-surface size stored in `alpha0_star` is evaluated from the constant-$\boldsymbol{\eta}$ image-point relation

$$\alpha_0^* = \frac{p'}{(g^*)^{1/m}}, \qquad g^* = 1 - \frac{(\boldsymbol{\eta} - \boldsymbol{b}) : (\boldsymbol{\eta} - \boldsymbol{b})}{\zeta^2}. \tag{24a}$$

1.10.1 How the mapping is done in the implementation

The standalone and UMAT implementations use the current trial stress-ratio direction $\boldsymbol{\eta}_{tr}$, not the start-of-step direction, to define the mapping ray. For a trial state that remains inside the bounding surface:

$$f_{tr} \leq 0 \qquad \text{and} \qquad f_n < 0,$$

the code evaluates

$$\zeta^2 = c^2(e, J_3^\eta) + \boldsymbol{b} : \boldsymbol{b} - 2\,\boldsymbol{\eta} : \boldsymbol{b}, \tag{24b}$$

$$A = (\boldsymbol{\eta} - \boldsymbol{b}) : (\boldsymbol{\eta} - \boldsymbol{b}), \qquad g^* = 1 - \frac{A}{\zeta^2}, \qquad 0 < g^* \leq 1, \tag{24c}$$

and then constructs the homothetic image-point size

$$\alpha_0^* = \frac{p'}{(g^*)^{1/m}}. \tag{24d}$$

For the current loading-surface size α', the mapped image mean stress is

$$p^I = \alpha'(g^*)^{1/m}, \tag{24e}$$

so the image stress used by the implementation is

$$\boldsymbol{\sigma}^I = p^I \boldsymbol{I} + p^I \boldsymbol{\eta}. \tag{24f}$$

The first time a given mapping phase becomes active, the code stores

$$\alpha_{0i} = \alpha_0^*, \tag{24g}$$

and thereafter forms

$$g_1 = \frac{\alpha' - \alpha_0^*}{\alpha' - \alpha_{0i}}, \qquad 0 \le g_1 \le 1, \tag{24h}$$

which is then used in Eqs. (23a) and (23b) to blend the image-point flow direction and hardening with the first-yield quantities.

So in practical terms:

- `enableOCMapping = 1`: the model treats an inside-surface loading increment as potentially plastic by computing an image point on the loading surface and evaluating $\boldsymbol{P}^I$ and $\langle H^I \rangle$ there.
- `enableOCMapping = 0`: the image-point construction above is skipped and the same inside-surface increment is accepted as elastic.

1.10.2 OCR used by the implementation

The MIT-S1 mini does not use a single universal `OCR` input during normal packaged runs. Instead, the OCR-like quantity appears in the initialization override through `initOCR`.

First define the implementation reference surface size

$$p_b'(e) = p_{\text{ref}}' \left(\frac{1}{e}\right)^{1/\rho_c}, \tag{2b}$$

and the corresponding anisotropy-corrected reference loading-surface size

$$\alpha_{\text{ref}}' = \frac{p_b'(e)}{1 + \boldsymbol{b} : \boldsymbol{b}/\alpha_0^2}. \tag{24i}$$

Then the initializer uses one of two rules:

$$\alpha' = \texttt{initOCR}\ p' \qquad \text{if } \texttt{initAlphaMode} = 1, \tag{24j}$$

$$\alpha' = \texttt{initOCR}\ \alpha_{\text{ref}}' \qquad \text{if } \texttt{initAlphaMode} = 0. \tag{24k}$$

Therefore the OCR-like ratio is interpreted as

$$\mathrm{OCR}_{\mathrm{mini}} = \frac{\alpha'}{p'} = \texttt{initOCR} \qquad \text{for } \texttt{initAlphaMode} = 1, \tag{24l}$$

whereas for `initAlphaMode=0` it is an LCC-referenced multiplier

$$\frac{\alpha'}{\alpha'_{\mathrm{ref}}} = \texttt{initOCR}. \tag{24m}$$

In the isotropic special case $\boldsymbol{b} = \boldsymbol{0}$, Eq. (24i) reduces to $\alpha'_{\mathrm{ref}} = p'_b$, so `initAlphaMode=0` becomes the classical isotropic OCR-style choice based on the ratio between the bounding-surface size and the LCC compression size.

1.11 Implementation Notes

- MIT-S1 uses explicit adaptive substepping for local stress integration.

1.12 Single-Point Validation

This page includes two K_0-undrained single-point benchmark reproductions based on Pestana & Whittle (1999). The figures compare the data reported in that paper with the corresponding FALCON MIT-S1 responses.

1.13 FALCON mini

The packaged mini tool id is `MITS1`. It lives under `mini_tools/MITS1`.

1.13.1 How to run

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool MITS1
--mini-input
/path/to/UMATLIB_FALCON/falcon_minis/MITS1/cases/triaxial_drained
```

Packaged simulation families:

Packaged case	Path	Purpose
Drained triaxial	`cases/triaxial_drained/input.txt`	Monotonic drained triaxial reference path.
Undrained triaxial	`cases/triaxial_undrained/input.txt`	Monotonic undrained triaxial reference path.

Packaged case	Path	Purpose
Cyclic drained strain-controlled	cases/cyclic_drained_strain/input.txt	Stabilized cyclic drained strain example.
Cyclic undrained strain-controlled	cases/cyclic_undrained_strain/input.txt	Stabilized cyclic undrained strain example.
Cyclic undrained q-controlled	cases/cyclic_undrained_q/input.txt	Mild cyclic undrained stress-controlled example.
Isotropic compression	cases/isotropic_compression/input.txt	Isotropic compression reference path for the bounding surface and LCC response.

1.13.2 Input syntax

`input.txt` uses whitespace-delimited `Key Value` pairs, one item per line, for example:

```
Mode triaxial_undrained
Control strain
OutputCSV triaxial_undrained_results.csv
Sigma3Init 500.0
VoidRatio0 0.80
```

The main driver selectors are `Mode` and `Control`. In practice, each packaged MIT-S1 case file contains:

- one loading-program selector (`Mode`)
- one control selector (`Control`)
- one output/driver-control block
- one loading-history block
- one initial-state block
- one MIT-S1 constitutive parameter block

`Mode` value	Typical `Control`	Meaning in the standalone mini
triaxial_drained	strain	Monotonic drained triaxial compression with a solved lateral strain response.
triaxial_undrained	strain	Monotonic undrained triaxial compression with zero total volumetric strain.

`Mode` value	Typical `Control`	Meaning in the standalone mini
`cyclic_drained_strain`	`strain`	Cyclic drained triaxial loading from a prescribed axial-strain waveform.
`cyclic_undrained_strain`	`strain`	Cyclic undrained triaxial loading from a prescribed axial-strain waveform.
`cyclic_undrained_q`	`q`	Cyclic undrained triaxial loading from a prescribed deviatoric-stress waveform.
`isotropic_compression`	`strain`	Isotropic compression using prescribed volumetric strain increments.

Mini inputs used by the packaged cases:
Driver and loading controls:

Input key	Used by	Required / choices / defaults	Meaning
`Mode`	all cases	Required; choices `triaxial_drained`, `triaxial_undrained`, `cyclic_drained_strain`, `cyclic_undrained_strain`, `cyclic_undrained_q`, `isotropic_compression`	Selects the standalone loading program.
`Control`	all triaxial cases	Required; choices `strain` or `q` depending on mode	Chooses whether the cyclic/monotonic branch is strain-driven or `q`-driven.

Input key	Used by	Required / choices / defaults	Meaning
`Waveform`	cyclic cases	Required for cyclic cases; packaged cases use `triangle`	Shape of the cyclic loading signal used by the standalone driver. The packaged cyclic cases use `triangle`.
`OutputCSV`	all cases	Optional; packaged cases set it explicitly	Name of the CSV written in the case directory.
`writeSubsteps`	all cases	Optional; choices `0/1` or `false/true`; packaged cases enable it	If enabled, substep-level rows are also written to the output CSV.
`ConditionInitialState`	all cases	Optional; choices `false/true`; packaged cases use `true`	If `true`, MIT-S1 internal state variables are conditioned once from the prescribed initial state before loading begins.
`nSteps`	monotonic and isotropic cases	Required for monotonic and isotropic modes	Number of loading increments in the monotonic or isotropic path.
`dEpsAxial`	monotonic triaxial cases	Required for monotonic triaxial modes	Imposed axial strain increment for monotonic triaxial loading. Compression is negative in the packaged driver inputs.
`dEpsV`	isotropic compression	Required for `Mode = isotropic_compression`	Imposed volumetric strain increment for isotropic loading.
`nCycles`, `stepsPerCycle`	cyclic cases	Required for cyclic modes	Number of cycles and number of driver steps per cycle.

Input key	Used by	Required / choices / defaults	Meaning
`epsAxialMean`, `epsAxialAmp`	cyclic strain-controlled cases	Required for cyclic strain-controlled modes	Mean and amplitude of the axial-strain waveform.
`qMean`, `qAmp`	cyclic q-controlled case	Required for `Mode = cyclic_undrained_q`	Mean and amplitude of the deviatoric stress waveform.

Initial-state inputs:

Input key	Used by	Required / choices / defaults	Meaning
`Sigma3Init`	all triaxial and isotropic cases	Required in packaged cases	Initial confining stress used to construct the axisymmetric starting stress state.
`K0Init`	all cases	Required in packaged cases	Initial lateral-to-vertical stress ratio used when the starting stress tensor is assembled.
`VoidRatio0`	all cases	Required in packaged cases	Initial void ratio.
`AlphaFactor`	strain-controlled cases	Optional; packaged strain-controlled cases use `0.0`	Optional scale factor used when conditioning the initial size of the MIT-S1 bounding surface. The packaged strain-controlled cases use `0.0`.

MIT-S1 constitutive parameters:

Input key	Used by	Required / choices / defaults	Meaning
`Pa`	all cases	Required in packaged cases	Reference pressure.
`rho_c`, `p_ref`, `theta`	all cases	Required in packaged cases	Parameters governing the limiting compression curve and the distance to the LCC.
`Cb`, `K0NC`, `nu0`, `omega`, `omega_s`	all cases	Required in packaged cases	Elasticity and stress-reversal stiffness controls.
`phi_cs`, `phi_mr`, `p`, `m`, `psi`	all cases	Required in packaged cases	Failure-surface, density-dependence, bounding-surface shape, and anisotropy-rate parameters.
`D`, `r`, `h`	all cases	Required in packaged cases	Hysteretic swelling and overconsolidated mapping controls.
`P_min`, `FTOL`, `stressRelTol`, `max Substeps`	all cases	Optional safeguards/tolerances; packaged cases set them explicitly	Numerical safeguards and local-integration tolerances.
`integrationScheme`	most cases	Optional; packaged cases set it explicitly where needed	Selects the local stress-update algorithm used by the UMAT.

Input key	Used by	Required / choices / defaults	Meaning
`enableOCMapping`	most cases	Optional; choices `0`/`1`; default `1` in the standalone mini	Turns the inside-surface overconsolidated mapping on or off. With `1`, overconsolidated interior loading can still produce plastic response through mapping to an image point on the loading surface. With `0`, interior loading is treated as elastic until the bounding surface is reached directly.

1.13.3 Hydromechanical assumptions

The packaged MIT-S1 mini is an effective-stress mechanical driver:

- it does not include an intrinsic unsaturated constitutive law
- it does not solve a retention relation or a suction-dependent hardening law
- undrained behaviour is represented mechanically through zero total volumetric strain
- drained behaviour is represented mechanically through the lateral-stress constraint

So unlike GCC, the key richness in the MIT-S1 mini is not unsaturated coupling but the combination of:

- bounding-surface plasticity
- anisotropy evolution through `b_ij`
- stress-reversal-point memory
- hysteretic elastic degradation
- optional overconsolidated interior mapping

The packaged cases keep `ConditionInitialState true`, so the driver conditions the internal state from the prescribed initial stress and void ratio before loading begins. That is usually the safer way to run the MIT-S1 mini unless you are intentionally testing a hand-built custom state.

Meaning of `enableOCMapping` This switch matters mainly for overconsolidated states, where the current stress point can lie strictly inside the bounding surface.

- `enableOCMapping = 1`: the UMAT applies the MIT-S1 overconsolidated mapping step. For an interior loading increment, it constructs an image point on the loading surface along the current stress-ratio direction and uses that mapped point to evaluate plastic loading and hardening. This is the usual choice when you want the model to show gradual plasticity inside the bounding surface rather than a long purely elastic plateau.
- `enableOCMapping = 0`: the mapping step is skipped. If the trial state stays inside the bounding surface, that increment is accepted as elastic. Plastic flow then begins only when the stress path reaches the surface itself.

In practical terms, keep `enableOCMapping = 1` for the packaged OC-style MIT-S1 examples unless you are doing a debugging comparison or you intentionally want to suppress interior plasticity.

1.13.4 Sample input

Monotonic drained triaxial example Path: mini_tools/MITS1/cases/triaxial_drained/input.txt

```
Mode triaxial_drained
Control strain
OutputCSV triaxial_drained_results.csv
writeSubsteps 1
ConditionInitialState true

nSteps 250
dEpsAxial -1e-4

Sigma3Init 500.0
K0Init 1.0
VoidRatio0 0.80
AlphaFactor 0.0

Pa 100.0
rho_c 0.25
p_ref 100.0
theta 0.20
Cb 750.0
K0NC 0.50
nu0 0.25
omega 1.25
omega_s 4.0
phi_cs 32.0
```

```
phi_mr 30.0
p 2.0
m 0.8
psi 30.0
D 0.0
r 1.0
h 0.0
P_min 1e-9
FTOL 1e-4
stressRelTol 1e-5
maxSubsteps 500
integrationScheme 0
enableOCMapping 1
```

This is the monotonic drained reference path for the packaged MIT-S1 mini. It is the cleanest case for seeing the combined effect of bounding-surface hardening and void-ratio evolution without cyclic reversal effects.

Monotonic undrained triaxial example Path: mini_tools/MITS1/cases/triaxial_undrained/input.txt

This packaged case uses the same parameter set and initial state as the drained reference but switches the loading constraint to undrained constant-volume triaxial compression. It is therefore the direct companion case for comparing drained and undrained MIT-S1 stress paths.

Cyclic drained strain-controlled example Path: mini_tools/MITS1/cases/cyclic_drained_strain/input.txt

```
Mode cyclic_drained_strain
Control strain
Waveform triangle
OutputCSV cyclic_drained_strain_results.csv
writeSubsteps 1
ConditionInitialState true

nCycles 1
stepsPerCycle 60
epsAxialMean 0.0
epsAxialAmp -0.0025

Sigma3Init 300.0
K0Init 1.0
```

```
VoidRatio0 0.80
AlphaFactor 0.0
```

This packaged cyclic drained case is intentionally modest rather than aggressive. It is designed to show a stable cyclic drained loop in the standalone mini without turning the example into a long-running research benchmark.

Cyclic undrained strain-controlled example Path: `mini_tools/MITS1/cases/cyclic_undrained_strain/input.txt`

This packaged case uses the same style of triangular axial-strain waveform but under the undrained constraint. It is the main packaged example for seeing how MIT-S1 evolves effective stress and the mapped bounding-surface state during cyclic undrained strain loading.

Cyclic undrained q-controlled example Path: `mini_tools/MITS1/cases/cyclic_undrained_q/input.txt`


```
Mode cyclic_undrained_q
Control q
Waveform triangle
OutputCSV cyclic_undrained_q_results.csv
writeSubsteps 0
ConditionInitialState true

stepsPerCycle 100
nCycles 10
qMean 10.0
qAmp 15.0

Sigma3Init 100.0
K0Init 1.0
VoidRatio0 0.90
```

This is the mild stress-controlled cyclic example shipped with the MIT-S1 mini. It is intended as a stable packaged case for exploring cyclic effective-stress reduction, not as a strong liquefaction benchmark.

Isotropic compression example Path: `mini_tools/MITS1/cases/isotropic_compression/input.txt`

This packaged case is the simplest way to inspect the LCC-driven part of MIT-S1 in isolation because `q = 0` throughout the run and the response is governed by isotropic compression only.

1.13.5 Output files and columns

Each case writes its own CSV, for example `triaxial_drained_results.csv`, `cyclic_drained_strain_results.csv`, or `cyclic_undrained_q_results.csv`.

Output file	Produced by	Main use
`*_results.csv`	all cases	Main MIT-S1 history file for the selected loading path.

Primary output columns:

Output column	Meaning
`step`, `substep`, `cycle`, `phase`, `mode`	Driver bookkeeping columns identifying where each saved row lies within the loading history.
`p_kpa`, `q_kpa`, `sigma1_kpa`, `sigma3_kpa`	Stress-path measures and triaxial principal stresses.
`eps_a_pct`, `eps_v_pct`, `e`	Axial strain, volumetric strain, and void ratio.
`alpha_kpa`, `pb_kpa`	Current bounding-surface size and LCC-related pressure measure.
`delta`, `delta_b`	Distance measures used in the MIT-S1 compression and bounding-surface formulation.
`alpha0_star_kpa`, `alpha0_i_kpa`, `g1_map`	Overconsolidated mapping variables.
`p_srp_kpa`, `mu`, `srp_has_reversal`	Stress-reversal-point memory variables.
`b_xx` to `b_xy`	Components of the anisotropy tensor.
`target1`, `target2`, `res1`, `res2`	Driver target and residual columns, useful mainly for checking the cyclic path-control solve.

When reading the packaged MIT-S1 outputs, a practical workflow is:

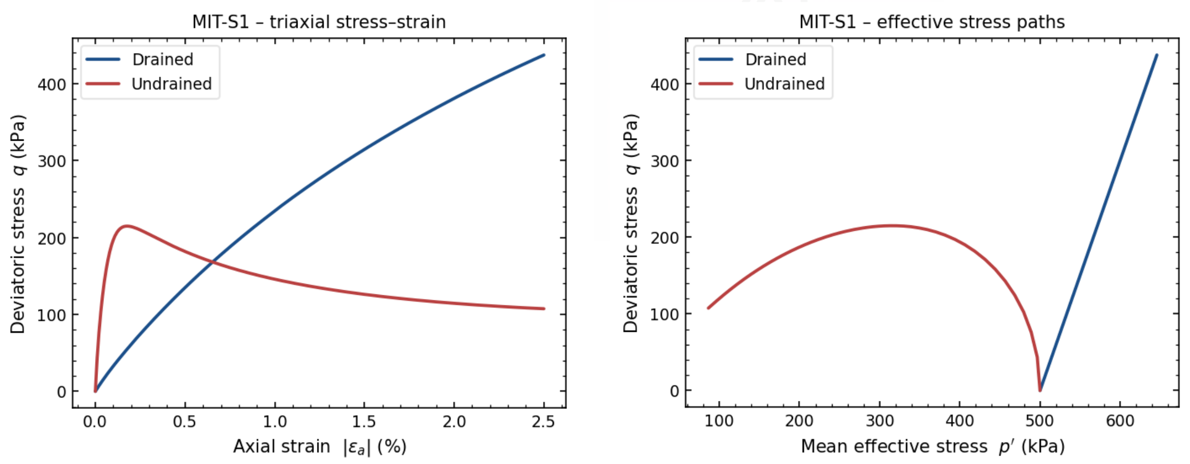
1. inspect `p_kpa`, `q_kpa`, `eps_a_pct`, and `eps_v_pct` first to understand the loading path
2. inspect `e`, `alpha_kpa`, and `pb_kpa` next to see how void ratio and hardening evolve
3. inspect `delta`, `delta_b`, `alpha0_star_kpa`, `alpha0_i_kpa`, and `g1_map` if you want to follow the bounding-surface mapping explicitly
4. inspect `p_srp_kpa`, `mu`, and `b_ij` if you want to study reversal memory and anisotropy evolution
5. inspect `target1`, `target2`, `res1`, and `res2` only when checking the quality of the driver path-control solve

The packaged cyclic strain-controlled cases are intentionally conservative so the standalone mini remains stable in routine use. The plots in the next section are generated directly from these packaged case CSVs.

1.14 Results

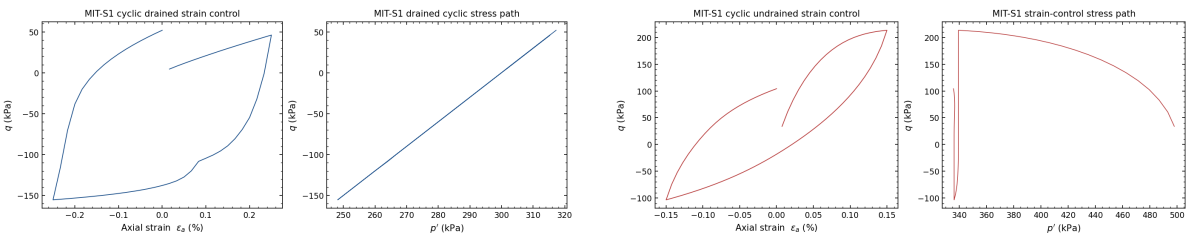
The plots below are produced directly from the bundled FALCON mini case inputs under `mini_tools/MITS1/cases`. The packaged MIT-S1 examples use a common Boston Blue Clay-style parameter family so that the effect of the loading program can be seen more clearly than the effect of changing calibration.

1.14.1 Monotonic drained and undrained triaxial response



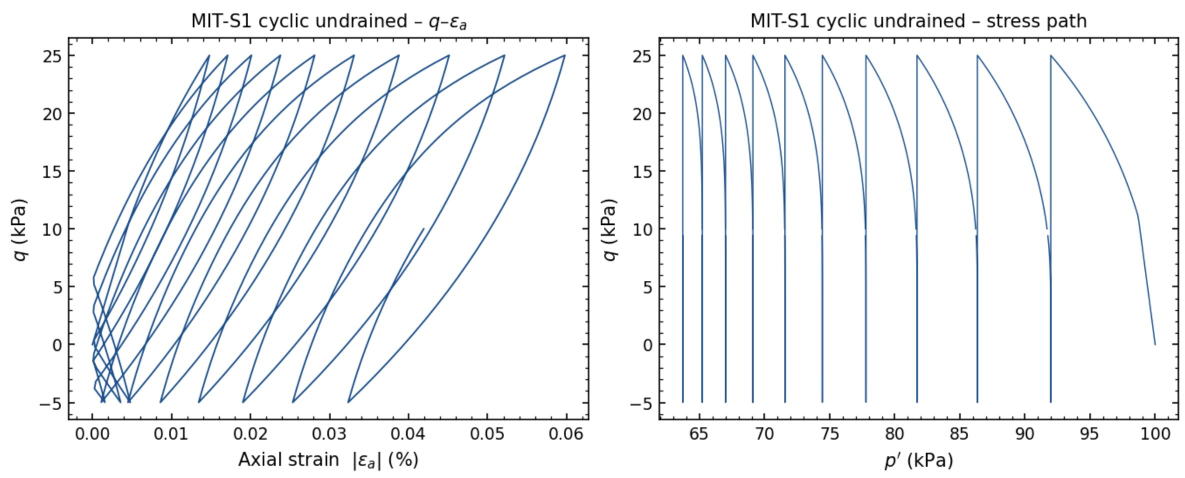
Bundled cases `cases/triaxial_drained/input.txt` and `cases/triaxial_undrained/input.txt`. Left: monotonic drained and undrained `q-\varepsilon_a` response. Right: the corresponding `p'-q` stress paths.

1.14.2 Cyclic strain-controlled examples



Bundled cases cases/cyclic_drained_strain/input.txt and cases/cyclic_undrained_strain/input.txt. These are the packaged stabilized cyclic strain examples. The drained case shows the solved constant-confining cyclic path, while the undrained case shows the corresponding cyclic effective-stress loop under zero total volumetric strain.

1.14.3 Cyclic undrained q-controlled example



Bundled case cases/cyclic_undrained_q/input.txt. This figure shows the mild stress-controlled cyclic response shipped with the mini.

1.14.4 Isotropic compression reference

The packaged isotropic case cases/isotropic_compression/input.txt is included primarily as a numerical and constitutive reference path. It is not repeated as a large figure here because the triaxial and cyclic cases are usually more informative for routine MIT-S1 use, but the output CSV is valuable when checking the LCC response and isotropic hardening evolution in isolation.

1.15 Additional benchmark reproductions

1.15.1 Prescribed K_0 consolidation effect on undrained clay response

The first reproduction follows the clay benchmark reported in Pestana & Whittle (1999) Figure 13. The figure compares the paper data with the FALCON MIT-S1 response for the same parameter sets.

Use pestana1999_fig13_ck0_strength_matching.txt as the starting deck and vary `phi_mr`, `m`, and the sign of `dEpsAxial` to recover the different compression/extension branches.

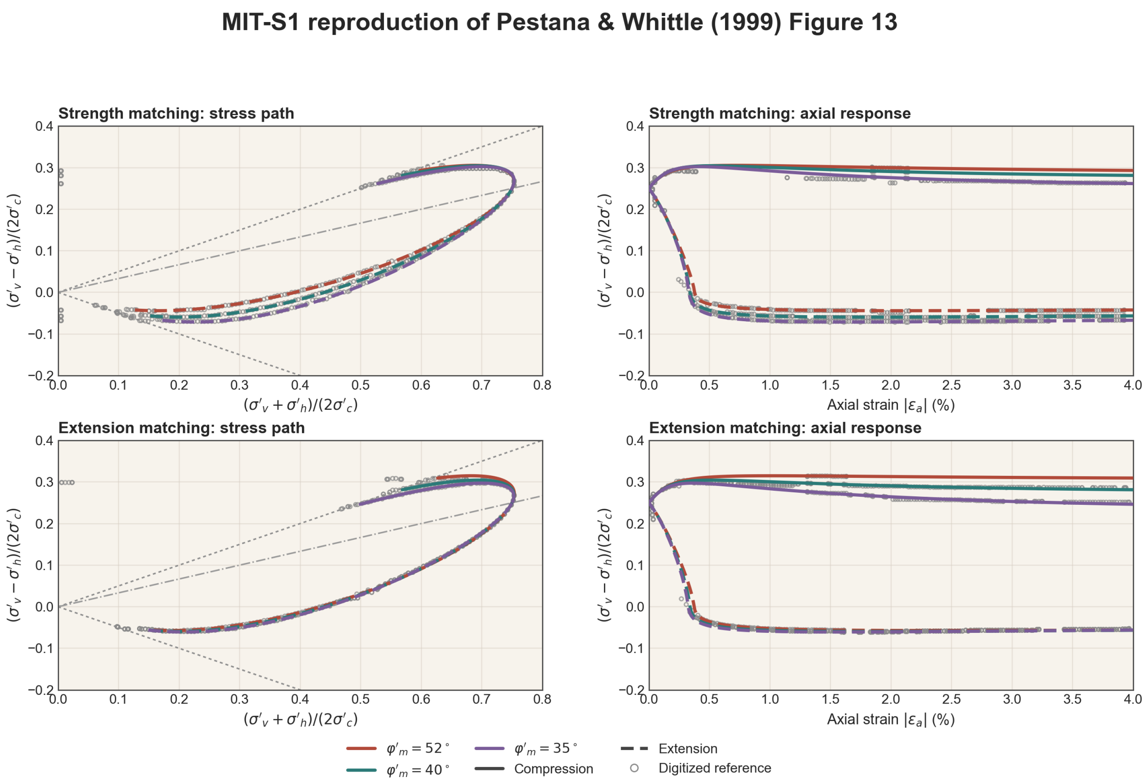


Figure 1: MIT-S1 alternate reproduction of Pestana and Whittle 1999 Figure 13

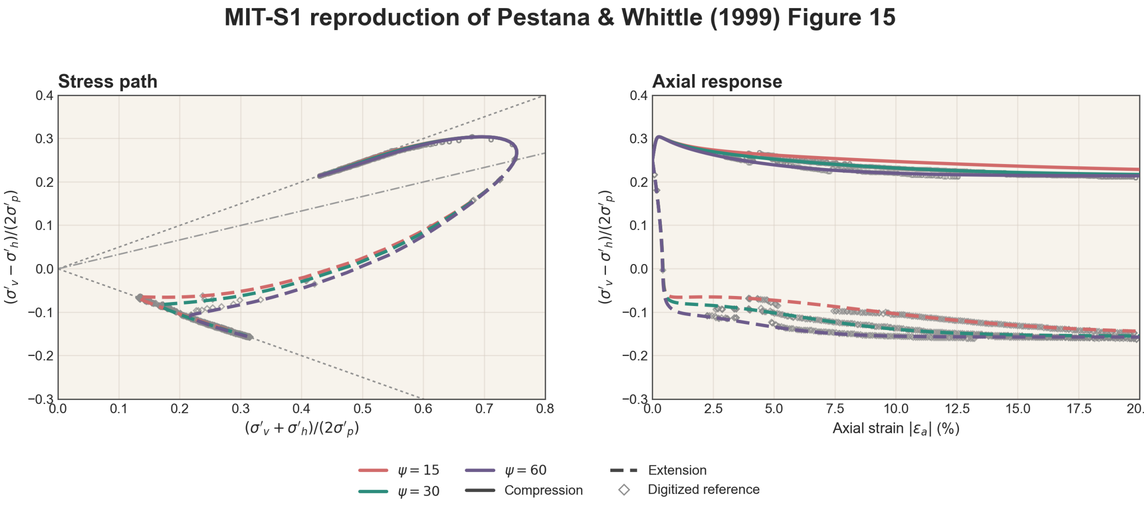


Figure 2: MIT-S1 alternate reproduction of Pestana and Whittle 1999 Figure 15

1.15.2 Anisotropy-rate effect in K_0-consolidated undrained clay

The second reproduction follows Pestana & Whittle (1999) Figure 15, which isolates the effect of the anisotropy-rate parameter ψ for K_0-consolidated clay. The figure compares the paper data with the FALCON MIT-S1 response.

The sample deck `pestana1999_fig15_ck0_psi30.txt` reproduces the middle $\psi = 30$ case; sweep `psi = 15, 30, 60` to rebuild the full family.

1.16 Applications and limitations

- Best suited to soils where anisotropy, bounding-surface response, and stress-reversal effects are important.
- Appropriate for uncoupled and effective-stress-based coupled analyses. Not an intrinsic unsaturated constitutive law.
- Not a replacement for dedicated liquefaction-specific sand models without case-specific verification.

1.17 References

- Pestana, J. M., & Whittle, A. J. (1999). *Formulation of a unified constitutive model for clays and sands. International Journal for Numerical and Analytical Methods in Geomechanics*, 23, 1215–1243.