Unsaturated Generalized Cam-Clay (GCC) Model

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1   Unsaturated Generalized Cam-Clay (GCC) Model

The GCC formulation follows Sheng et al. (2000) for the generalized yield surface, Gallipoli et al. (2003) and Borja (2004) for the suction-dependent state variable, and Ghorbani et al. (2018) for the hysteretic soil-water retention description. The model unifies saturated Modified Cam-Clay behaviour with suction-stiffened responses and is compatible with fully coupled hydro-mechanical finite elements.

1.1   Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/GCCModel.cpp path/to/GCCModel.hpp Mechanical \
    Phi=30 Lambda=0.15 Kappa=0.03 Nu=0.25 Alpha=0.8 Beta_prime=1.0 \
    OCRControlled=1 DefaultIsoHardening=500 v_N=2.0 \
    P_min=0.1 patm=100 STOL=1e-5 FTOL=1e-6 LTOL=1e-6 \
    CustomVariable=IsotropicHardening,Delta_vN
```

For readability, this example is wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

Use the parameter names shown in the tables below.

1.2   Material parameters

Table 1. Material parameters and their descriptions

Symbol	Keyword in input	Units	Required	Description
ϕ'	Phi	°	✓	Critical-state friction angle.
λ	Lambda	–	✓	Virgin compression index (slope of NCL in v–$\ln p'$ space).
κ	Kappa	–	✓	Swelling/reloading index.
v	Nu	–	✓	Poisson's ratio.

Symbol	Keyword in input	Units	Required	Description
α	Alpha	–	✓	Ratio M_c/M_e fixing Lode-angle dependence.
β'	Beta_prime	–	✓	Wet-side eccentricity ($\beta' \leq 1$ recovers MCC when $\beta' = 1$).
β	Beta	–	✗	Optional eccentricity parameter (defaults to 1.0 when omitted).
c_1	c1	–	✗	Optional suction-coupling coefficient (defaults to 0.0 when omitted).
c_2	c2	–	✗	Optional suction-coupling coefficient (defaults to 0.0 when omitted).
v_N	v_N	–	✓	Specific volume at $p' = 1$ kPa on the saturated NCL.
$P_{\min}$	P_min	stress	✓	Lower bound applied inside the elastic bulk modulus.
p_{atm}	patm	stress	✓	Normalising pressure appearing in ζ.
STOL	STOL	–	✓	Stress integration tolerance for sub-stepping.
FTOL	FTOL	–	✓	Yield-surface tolerance for return mapping / drift correction.
LTOL	LTOL	–	✓	Load–unload detection tolerance.

Symbol	Keyword in input	Units	Required	Description
a_0	`DefaultIso Hardening`	stress	✓	Minimum $\sigma_{mc0}^{\text{sat}}$ used in initialization.
OCRControlled	`OCRControlled`	–	✓	Initialization/conditioning mode selector (see below).
OBN	`OBN`	–	×	Optional overburden modifier (defaults to 0.0 when omitted).

1.3 Custom state variables

This UMAT uses custom state variables to store hardening-related quantities. Declare them using `CustomVariable=` in the `@UMAT:` line.

Name	Required	Meaning
`Isotropic Hardening`	✓	The saturated cap size / isotropic hardening variable.
`Delta_vN`	✓	Increment applied to `v_N` during conditioning (typically initialized to 0).
`OCR`	×	Required only for OCR-based conditioning modes; ignored otherwise.

Optional diagnostics (only if you include them in `CustomVariable=`):

- `PlasticStrainIncXX`, `PlasticStrainIncYY`, `PlasticStrainIncZZ`, `PlasticStrainIncZY`, `PlasticStrainIncZX`, `PlasticStrainIncXY` (declare these via `CustomVariable=` if you want them tracked and available for output)
- `NegDFailureFlag`, `BracketingFailureFlag`, `DriftFailureFlag`

1.4 Effective stress and suction-enhanced bonding

1.4.1 Net and matric quantities

Net stress and matric suction follow the tension-positive sign convention:

$$\sigma_{\text{net}} = \sigma + p_a \mathbf{I}, \qquad p_c = p_a - p_w \tag{1}$$

1.4.2 Effective stress with saturation weighting

Bishop's effective stress, written directly in terms of pore pressures and the degree of saturation, reads

$$\sigma' = \sigma - S_w\, p_w\, \mathbf{I} - (1 - S_w)\, p_a\, \mathbf{I} \tag{2}$$

The suction stress is obtained from the enriched Bishop coefficient $\chi(S_w)$,

> Note: The $\chi(S_w)$ weighting (and its parameters such as $\beta_{\chi,1}$ and $\beta_{\chi,2}$) is configured via your selected effective stress model, not via the GCC UMAT mechanical parameter list.

$$\chi(S_w) = S_w^{\left(\frac{\beta_{\chi,1}}{S_w^{\beta_{\chi,2}}}\right)}, \qquad \sigma_{\text{suc}} = \chi\, p_c \tag{3}$$

which recovers the classical $\chi = S_w$ when $\beta_{\chi,1} = 1$ and $\beta_{\chi,2} = 0$ used in the original model. Equations (1)–(3) apply throughout the mechanical formulation and in the suction-coupling terms.

1.5 Unsaturated state variable and cap hardening

The bonding variable ζ couples suction and saturation to the mechanical hardening law (Borja, 2004):

$$\zeta = \left(1 + \frac{p_c/p_{\text{atm}}}{10.7 + 2.4\, p_c/p_{\text{atm}}}\right)(1 - S_w) \tag{4}$$

Gallipoli et al. (2003) linked the current and saturated void ratios at the same stress level through

$$\frac{e}{e_{\text{sat}}} = 1 - c_1\,(1 - \exp(c_2 \zeta)) \tag{5}$$

with the corrected coefficient

$$\bar{c}_1 = c_1 \frac{e_{\text{sat}}}{1 + e_{\text{sat}}} \tag{6}$$

so that the ζ-dependent compressibility factor becomes

$$c_\zeta = 1 - \bar{c}_1\,(1 - \exp(c_2 \zeta)) = \frac{v}{v_{\text{sat}}} \tag{7}$$
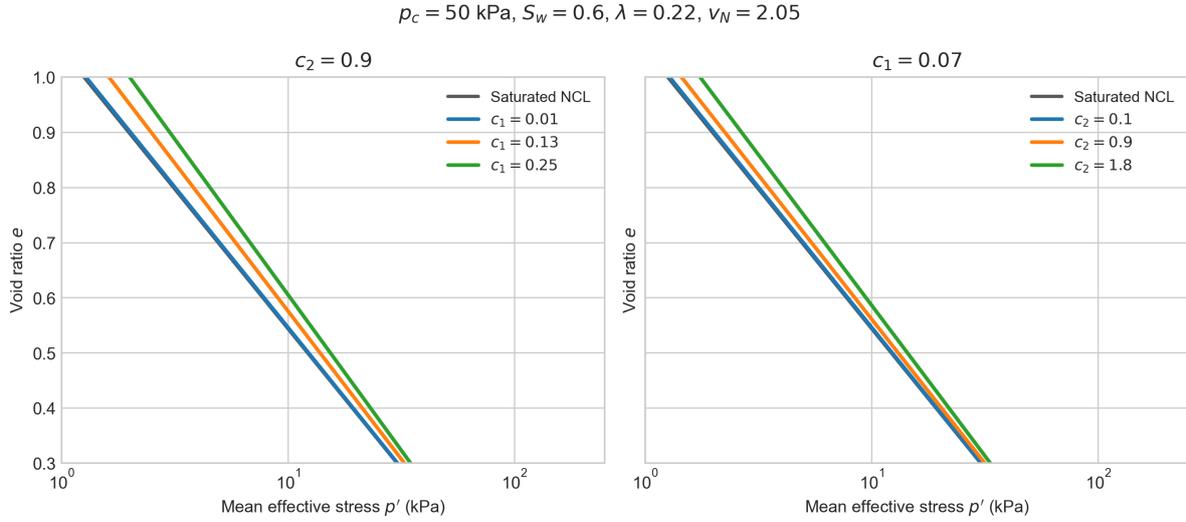
Figure 1: Influence of c_1 and c_2 on the unsaturated normal compression line

The unsaturated cap parameter, which sets the size of the yield surface, is related to the saturated value through

$$\sigma_{mc0}^{\text{unsat}} = \left(\sigma_{mc0}^{\text{sat}}\right)^{b_\zeta} \exp(a_\zeta) \tag{8}$$

where

$$a_\zeta = \frac{v_N(c_\zeta - 1)}{\lambda c_\zeta - \kappa}, \qquad b_\zeta = \frac{\lambda - \kappa}{\lambda c_\zeta - \kappa} \tag{9}$$

The specific volume at $p' = 1$ kPa, v_N, controls the intercept of the normal compression line (NCL), while λ and κ govern its slope and elastic unloading stiffness. Increasing c_1 deepens the intercept reduction, whereas c_2 amplifies the suction sensitivity, as illustrated in Figure 1. The plotted curves follow Equation (5) directly in e–p' space using a logarithmic stress axis between 1 and 250 kPa at a fixed suction state ($p_c = 50$ kPa, $S_w = 0.6$).

Figure 1. Stylised NCL shifts relative to the saturated line for different c_1 (left) and c_2 (right); curves are generated directly from Equation (5) in e–p' space with a log-scaled stress axis covering 1–250 kPa.

1.6   Yield surface and plastic potential

The generalized Cam-Clay yield function in p'–q–θ space reads

$$f = \frac{1}{\beta_{\text{side}}^2} \left(1 + \frac{(1 + \beta')p'}{\sigma_{mc0}^{\text{unsat}}}\right)^2 + \left(\frac{(1 + \beta')q\, F_\theta}{M\, \sigma_{mc0}^{\text{unsat}}}\right)^2 - 1 = 0 \tag{10}$$

with the Lode-angle modifier

$$F_\theta = \left[\frac{1 + \alpha^4 - (1-\alpha^4)R_\theta}{2\alpha^4}\right]^{1/4} \tag{11}$$

and the third invariant ratio

$$R_\theta = -\frac{3\sqrt{3}J_3}{2J_2^{3/2}} \tag{12}$$

Here $M = 6\sin\phi'/(3 - \sin\phi')$ is the critical-state slope, and $\sigma_{mc0}^{\text{unsat}}$ sets the cap size. The eccentricity switches between the dry and wet sides through

$$\beta_{\text{side}} = \begin{cases} 1, & \text{(dry side) or } \beta' = 1 \\ \beta', & \text{(wet side) and } \beta' < 1 \end{cases} \tag{10a}$$

Equation (10a) therefore recovers the smooth Modified Cam-Clay ellipse ($\beta_{\text{side}} = 1$) on the dry side, whereas the wet side adopts the rounded GCC curvature ($\beta_{\text{side}} = \beta' \leq 1$). The plastic potential shares the same structure (associated flow) unless a dilatancy correction is introduced. Figure 2 compares the two sides and highlights that only the wet branch is modified when $\beta' < 1$; the horizontal axis follows the tension-positive convention (compression at negative p') and only the compressive branch $q = Mp'$ is shown, with the vertical axis inverted to emphasise the negative q portion.

Figure 2. Numerical comparison of the MCC ($\beta' = 1$) and GCC ($\beta' = 0.45$) surfaces

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1.7 Elasticity, consistency, and tangents

The elastic moduli follow isotropic linear elasticity with stress-dependent bulk stiffness:

$$K = \frac{v \cdot \max(P_{\min}, p')}{\kappa}, \qquad v = 1 + e \tag{13}$$

$$G = \frac{3(1 - 2\nu)}{2(1 + \nu)}K \tag{14}$$

$$\mathbf{D}^e = \begin{bmatrix} \lambda_e + 2\mu & \lambda_e & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e + 2\mu & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e & \lambda_e + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \lambda_e = K - \tfrac{2}{3}G, \ \mu = G \tag{15}$$

Loading on the yield surface enforces the consistency condition

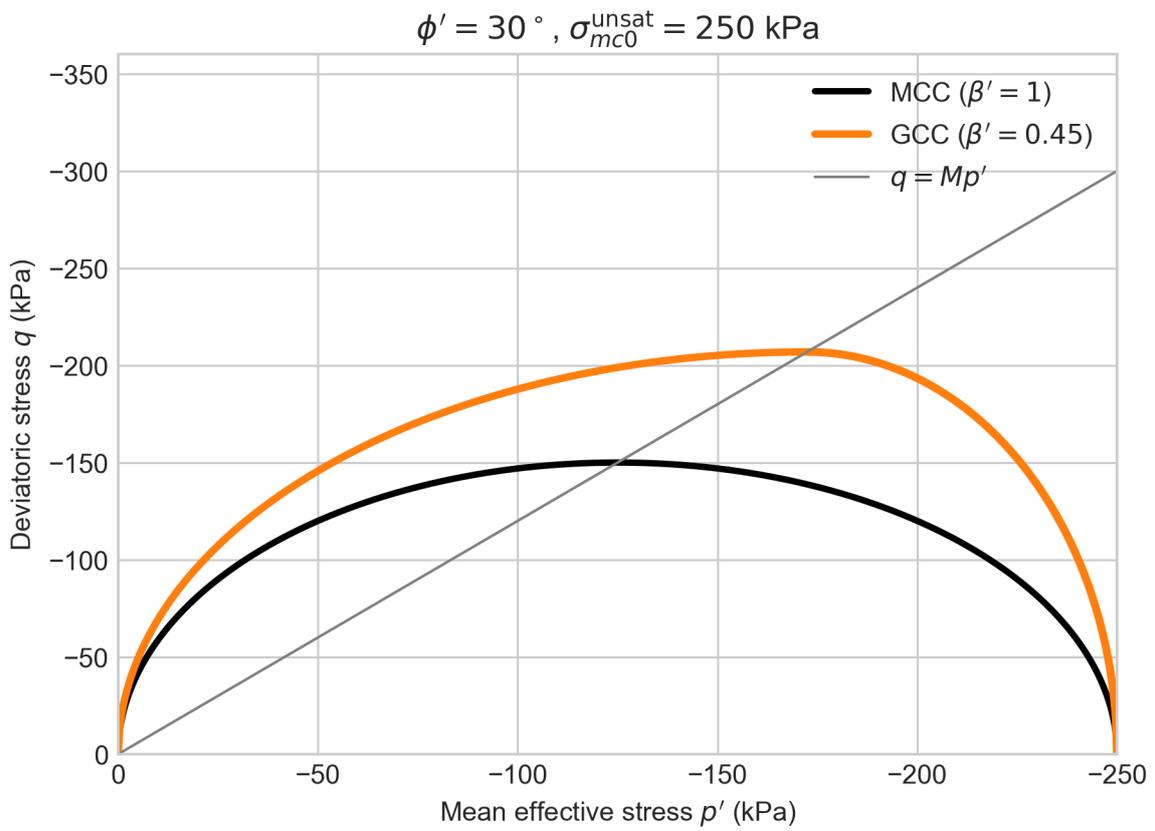$$\phi' = 30°, \sigma_{mc0}^{\text{unsat}} = 250 \text{ kPa}$$

Figure 2: Smooth MCC and GCC yield surfaces in $p'-q$ space

$$\frac{\partial f}{\partial \boldsymbol{\sigma}'} : d\boldsymbol{\sigma}' + \frac{\partial f}{\partial \zeta} d\zeta + \frac{\partial f}{\partial \sigma_{mc0}^{\text{unsat}}} d\sigma_{mc0}^{\text{unsat}} = 0 \tag{16}$$

with the elastic–plastic decomposition

$$d\boldsymbol{\sigma}' = \mathbf{D}^e \left(d\varepsilon - d\lambda \frac{\partial g}{\partial \boldsymbol{\sigma}'} \right) \tag{17}$$

yielding the plastic multiplier

$$d\lambda = \frac{\dfrac{\partial f}{\partial \boldsymbol{\sigma}'} : \mathbf{D}^e d\varepsilon + \dfrac{\partial f}{\partial \zeta} d\zeta}{K_p + \dfrac{\partial f}{\partial \boldsymbol{\sigma}'} : \mathbf{D}^e : \dfrac{\partial g}{\partial \boldsymbol{\sigma}'}} \tag{18}$$

where the hardening modulus is

$$K_p = -\frac{\partial f}{\partial \sigma_{mc0}^{\text{unsat}}} \cdot B_{\text{iso}}, \qquad B_{\text{iso}} = \frac{\partial g}{\partial p'} \frac{\partial \sigma_{mc0}^{\text{unsat}}}{\partial \varepsilon_v^p} \tag{19}$$

The elastoplastic constitutive update including suction coupling is

$$d\boldsymbol{\sigma}' = \mathbf{D}_{ep} \, d\varepsilon + \mathbf{S}_{ep} \, dp_c \tag{20}$$

with

$$\mathbf{D}_{ep} = \mathbf{D}^e - \frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}'} \otimes \frac{\partial f}{\partial \boldsymbol{\sigma}'} \mathbf{D}^e}{K_p + \frac{\partial f}{\partial \boldsymbol{\sigma}'} : \mathbf{D}^e : \frac{\partial g}{\partial \boldsymbol{\sigma}'}} + \mathbf{S}_{ep}^{\varepsilon_v} \, \boldsymbol{m}^T \tag{21}$$

$$\mathbf{S}_{ep} = -\frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}'} \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial p_c}}{K_p + \frac{\partial f}{\partial \boldsymbol{\sigma}'} : \mathbf{D}^e : \frac{\partial g}{\partial \boldsymbol{\sigma}'}} \tag{22}$$

$$\mathbf{S}_{ep}^{\varepsilon_v} = -\frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}'} \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial \varepsilon_v}}{K_p + \frac{\partial f}{\partial \boldsymbol{\sigma}'} : \mathbf{D}^e : \frac{\partial g}{\partial \boldsymbol{\sigma}'}} \tag{23}$$

and $\boldsymbol{m} = [1, 1, 1, 0, 0, 0]^T$ is the volumetric projection vector. These consistent tangents feed directly into the finite-element stiffness and coupling matrices.

1.8   Hysteretic soil–water retention curve

The hysteretic soil–water retention formulation used by the GCC UMAT is documented in detail in `swrchys.md`. That note covers the modified suction definition, the van Genuchten main curves, the scanning-rule interpolation, and the void-ratio coupling that ties the hydraulic response back to the mechanical update.

1.9 Post-equilibrium state conditioning

After establishing stress equilibrium (e.g., following a geostatic initialization step), the model must ensure that the current stress state, void ratio, and hardening variables satisfy the consistency conditions of the elastoplastic framework. The `OCRControlled` flag selects how the initial cap stress (σ_{mc0}) and isotropic hardening variable are recovered from the equilibrium stress state to enforce compatibility between mechanical and hydraulic fields. Two branches are available:

- **OCRControlled = 1** – Used in **fully coupled unsaturated analyses**. A Picard-type iterative scheme maintains total stress constant while updating void ratio, saturation, and effective stress until convergence, ensuring consistency between the mechanical state and the hydraulic response.

- **OCRControlled $\neq$ 1** – Available in **coupled saturated** and **uncoupled analyses only**. The current stress state is placed on the yield surface, and a prescribed offset (`DefaultIso Hardening`, `OCR`) defines the distance to the yield surface while ensuring the void ratio and cap size remain compatible with the elastoplastic formulation.

1.9.1 State-fitted initialization – OCRControlled $\neq$ 1

Let the current effective stress invariants be p', q and F_θ the Lode-angle factor. The cap parameter on the saturated surface follows directly from the yield function,

$$f = \left(1 + \frac{(1 + \beta')p'}{\sigma_{mc0}^{\text{sat}}}\right)^2 + \left(\frac{(1 + \beta')q\,F_\theta}{M\,\sigma_{mc0}^{\text{sat}}}\right)^2 - 1 = 0 \tag{24}$$

with $Q = qF_\theta/M$. Two cases arise:

- **Dry side or** $\beta' = 1$

$$\sigma_{mc0}^{\text{sat}} = -\frac{p'^2 + Q^2}{p'} \tag{25}$$

- **Wet side with** $\beta' < 1$

$$\frac{1}{\beta'^2}\left(1 + \frac{p'}{\sigma_{mc0}^{\text{sat}}/(1 + \beta')}\right)^2 + \left(\frac{Q}{\sigma_{mc0}^{\text{sat}}/(1 + \beta')}\right)^2 = 1 \tag{26}$$

Introducing $x = (1 + \beta')/\sigma_{mc0}^{\text{sat}}$ gives

$$\left[p'^2 + \beta'^2 Q^2\right]x^2 + 2p'x + (1 - \beta'^2) = 0 \tag{27}$$

whose admissible solution is

$$\sigma_{mc0,-}^{\text{sat}} = \frac{(1 + \beta')\left(p'^2 + \beta'^2 Q^2\right)}{-p' - \beta'\sqrt{p'^2 - (1 - \beta'^2)Q^2}} \tag{28}$$

Once $\sigma_{mc0}^{\text{sat}}$ is known, the unsaturated cap follows from Equation (8). The OCR and default floor are applied via

$$\text{IsotropicHardening} = \max\left(\sigma_{mc0}^{\text{sat}} + \text{OCR} \times \text{DefaultIsoHardening}, \ \sigma_{mc0}^{\text{sat}}\right) \tag{30}$$

`IsotropicHardening` always stores the saturated isotropic hardening variable. Unsaturated hardening values are not directly stored in FALCON and are obtained from the hydraulic state internally by the GCC UMAT.

1.9.2 Unsaturated `OCRControlled = 1` – Picard iteration

Maintaining the total stress σ_{tot} ensures that any change in void ratio, saturation, or suction feeds back consistently into the effective stress. A Picard loop enforces this coupling:

1. Initialize the void ratio from the supplied state and record σ_{tot}.
2. Compute $\sigma_{mc0}^{\text{sat}}$ from Equations (24)–(28), apply the chosen offset (Equations (30)–(31)), and recover $\sigma_{mc0}^{\text{unsat}}$ from Equation (8).
3. Update the void ratio, saturation, and effective stress from the new cap size.
4. Repeat until

$$\frac{|e_{\text{new}} - e_{\text{old}}|}{|e_{\text{old}} + 1|} < 10^{-5} \tag{32}$$

or the iteration limit (50 by default) is reached.

Regardless of the initialization branch, the void ratio is updated from the elastic NCL so that the specific volume remains compatible with the initialized `IsotropicHardening`,

$$v = v_N + \kappa\left(\ln(\text{IsotropicHardening}) - \ln(-p')\right) - \lambda \ln(\text{IsotropicHardening}), \qquad e = v - 1 \tag{37}$$

1.10 FALCON mini

The packaged mini tool id is GCCUMAT. It lives under `mini_tools/GCCUMAT`.

1.10.1 How to run

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool GCCUMAT
--mini-input
/path/to/UMATLIB_FALCON/falcon_minis/GCCUMAT/cases/const_wcontent
```

Packaged simulation families:

Packaged case	Path	Purpose
Drained triaxial	cases/drained/input.txt	Saturated drained reference response.
Undrained triaxial	cases/undrained/input.txt	Saturated constant-volume reference path.
Constant-suction q/p_net	cases/qoverpnet/input.txt	Unsaturated triaxial loading at fixed suction.
Constant-p_net isotropic	cases/const_pnet/input.txt	Suction-driven isotropic loading with dp_net = 0.
Constant-water-content triaxial	cases/const_wcontent/input.txt	Unsaturated triaxial loading with coupled hydraulic evolution.

1.10.2 Input syntax

input.txt uses whitespace-delimited Key Value pairs, one item per line, for example:

```
Mode Drained
Phi 30
Lambda 0.077
PoreAirPressure 0.0
```

The main driver selector is Mode. The GCC standalone driver does not use name=value syntax in these mini case files, and it reads a flat sequence of keys rather than nested blocks. In practice, each packaged input file contains:

- one loading-program selector (Mode)
- one constitutive parameter block
- one hydraulic/retention block when unsaturated behaviour is active
- one initial-state block
- one loading-control block

Mode value	Meaning in the standalone mini	Hydraulic assumption / constraint
Drained	Saturated drained triaxial loading. The driver applies axial strain and solves a radial strain increment so the confining stress stays close to the target lateral stress.	Saturated reference path with `p_a = p_w = 0`, so `S_w = 1` and suction is zero.
Undrained	Saturated constant-volume triaxial loading.	The driver enforces the saturated constant-volume condition and keeps suction at zero.
QOverPnet	Unsaturated triaxial loading intended to follow an approximately constant `dq / dp_net = 3` path.	Suction is held constant while the retention law and $\chi(S_w)$ still control S_w, ζ, and the effective stress.
ConstPnet	Unsaturated isotropic suction-driven path with `dp_net = 0`.	The driver prescribes suction change and solves the volumetric strain increment needed to keep `p_net` constant.
ConstWContent	Unsaturated triaxial loading intended to follow the same `q/p_net` target family while holding water content constant.	Suction, saturation, and void ratio evolve together so that the water-content constraint is maintained during loading.

Mini inputs used by the packaged cases:
Constitutive parameters:

Input key	Used by	Required / choices / defaults	Meaning
Phi	all cases	Required in packaged cases	Critical-state friction angle entering the GCC stress ratio `M`.
Lambda	all cases	Required in packaged cases	Virgin compression slope of the normal compression line.

Input key	Used by	Required / choices / defaults	Meaning
Kappa	all cases	Required in packaged cases	Elastic swelling/reloading slope.
Nu	all cases	Required in packaged cases	Elastic Poisson ratio used to build the isotropic elastic stiffness.
Alpha	all cases	Required in packaged cases	Lode-angle dependence parameter for the generalized yield surface.
Beta	unsaturated examples	Optional; packaged unsaturated cases set it explicitly	Optional eccentricity parameter retained by the standalone driver.
Beta_prime	unsaturated examples	Required in packaged unsaturated cases	Wet-side eccentricity modifier controlling how far the surface departs from MCC.
c1, c2	unsaturated examples	Required in packaged unsaturated cases	Suction-coupling parameters that shift the unsaturated compression state relative to the saturated one.
v_N	all cases	Required in packaged cases	Specific-volume intercept of the saturated NCL at $p' = 1$.
P_min	all cases	Optional; pressure-floor default exists but packaged cases set it explicitly	Pressure floor used in the elastic bulk modulus to avoid collapse of tangent stiffness at very low p'.

Input key	Used by	Required / choices / defaults	Meaning
`patm`	all cases	Required in packaged cases	Atmospheric normalization pressure entering the GCC suction-dependent state variable.
`STOL, FTOL, LTOL`	all cases	Optional; packaged cases set them explicitly	Stress-integration, yield-drift, and load-detection tolerances used by the standalone driver and UMAT.
`OCRControlled, DefaultIso Hardening`	all cases	Optional switches/seed values; packaged cases set them explicitly	Initialization controls for how the hardening state is fitted to the starting stress state.

Hydraulic and effective-stress inputs:

Input key	Used by	Required / choices / defaults	Meaning
`VG_n`, `VG_m`	unsaturated examples	Required for packaged unsaturated cases	Main-curve shape parameters for the hysteretic retention model used by the mini.
`VG_omega_prime`	unsaturated examples	Required for packaged unsaturated cases	Void-ratio coupling exponent used in the retention update implemented by the standalone driver.
`Pd`, `Pw`	unsaturated examples	Required for packaged unsaturated cases	Drying and wetting pressure scales for the main SWRC branches.

Input key	Used by	Required / choices / defaults	Meaning
`b_w`, `b_d`, `b_sc`, `alpha_p_c`	`ConstPnet`, `Const WContent`	Required when suction evolves; not needed for constant-suction `QOverPnet`	Scanning-curve and branch-blending controls used when suction evolves during the run.
`beta_x1`, `beta_x2`	unsaturated examples	Required for packaged unsaturated cases	Parameters of the $\chi(S_w)$ effective-stress weighting law used in the GCC mini driver.

Initial stress, pore-pressure, and state inputs:

Input key	Used by	Required / choices / defaults	Meaning
`VoidRatio`	all cases	Required in packaged cases	Initial void ratio.
`StressXX`, `Stress YY`, `StressZZ`	all cases	Required in packaged cases	Initial total stress components. The packaged cases use isotropic initial stress states.
`PoreAirPressure`, `PoreWaterPressure`	all cases	Required in packaged cases	Initial pore-air and pore-water pressures. The driver computes suction from `s = p_a - p_w`, so `0.0` and `-10.0` mean an initial suction of `10 kPa`.
`Isotropic Hardening`, `OCR`, `Delta_vN`	all cases	Required in packaged cases	GCC state variables used to initialize the cap size and normal-compression-line adjustment.

Loading controls:

Input key	Used by	Required / choices / defaults	Meaning
dEpsAxial	Drained, Undrained, QOverPnet, ConstWContent	Required for triaxial modes	Imposed axial strain increment for the triaxial branches. Compression is negative in these packaged drivers.
dSuction	ConstPnet	Required for ConstPnet	Imposed suction increment for the isotropic constant-p_{net} branch.
nSteps	all cases	Required in packaged cases	Number of accepted driver steps. The total imposed path length is therefore nSteps × increment.

1.10.3 Hydromechanical assumptions

The GCC mini is more than a mechanical wrapper. The packaged unsaturated cases explicitly combine:

- Bishop-type effective stress with \chi(S_w) weighting
- the GCC suction-dependent state variable \zeta
- a hysteretic SWRC with main drying/wetting branches and scanning curves
- void-ratio coupling in the retention update

  For the packaged unsaturated examples:

- PoreAirPressure 0.0 is used as the atmospheric air-pressure reference
- initial suction is therefore set through negative PoreWaterPressure
- QOverPnet keeps suction fixed during loading
- ConstPnet prescribes suction change and solves the isotropic volumetric response needed to keep p_{net} constant
- ConstWContent updates suction, saturation, and void ratio together to preserve water content

  So unlike the Mohr mini, the GCC mini is an intrinsically unsaturated constitutive driver rather than a saturated mechanical law paired with an external effective-stress model.

1.10.4 Sample input

Saturated drained triaxial example   Path: `mini_tools/GCCUMAT/cases/drained/input.txt`

```
Mode Drained
Phi 30
Lambda 0.077
Nu 0.3
Kappa 0.0066
Alpha 0.77
OCRControlled 0
DefaultIsoHardening 500
v_N 1.788
P_min 0.1
patm 100.0
STOL 1e-7
FTOL 1e-6
LTOL 1e-6
VoidRatio 0.5
StressXX -100
StressYY -100
StressZZ -100
PoreAirPressure 0.0
PoreWaterPressure 0.0
IsotropicHardening 500
OCR 1.0
Delta_vN 0.0
dEpsAxial -0.0001
nSteps 2000
```

This is the saturated drained baseline path. It uses p_a = p_w = 0, so the hydraulic state stays at S_w = 1, suction = 0, and the response reduces to the saturated GCC mechanical law.

Saturated undrained triaxial example   Path: `mini_tools/GCCUMAT/cases/undrained/input.txt`

```
Mode Undrained
Phi 30
Lambda 0.077
Nu 0.3
Kappa 0.0066
```

```
Alpha 0.77
c1 0.0
c2 0.0
OCRControlled 0
DefaultIsoHardening 500
v_N 1.788
P_min 0.1
patm 100.0
STOL 1e-7
FTOL 1e-6
LTOL 1e-6
VoidRatio 0.5
StressXX -100
StressYY -100
StressZZ -100
PoreAirPressure 0.0
PoreWaterPressure 0.0
IsotropicHardening 500
OCR 1.0
Delta_vN 0.0
dEpsAxial -0.0001
nSteps 2000
```

This packaged case uses the same saturated parameter set as the drained baseline but switches the driver to the constant-volume branch. In the standalone mini, constant volume is enforced kinematically through exx = ezz = -0.5 eyy, so this case is the saturated undrained reference for the GCC driver rather than a suction-evolving path.

Constant-suction q/p_net example   Path: mini_tools/GCCUMAT/cases/qoverpnet/ input.txt

```
Mode QOverPnet
Phi 25.0
Lambda 0.11
Nu 0.30
Kappa 0.03
Alpha 0.77
Beta 1.0
Beta_prime 1.0
c1 0.2
c2 1.5
OCRControlled 0
DefaultIsoHardening 200
```

```
v_N 2.76
P_min 0.1
patm 100.0
STOL 1e-9
FTOL 1e-9
LTOL 1e-6
VG_n 2.5
VG_m 0.6
VG_omega_prime 0.2
Pd 0.05
Pw 0.1
beta_x1 1.0
beta_x2 0.0
VoidRatio 1.5
StressXX -100
StressYY -100
StressZZ -100
PoreAirPressure 0.0
PoreWaterPressure -10.0
IsotropicHardening 200
OCR 2.0
Delta_vN 0.0
dEpsAxial -1e-5
nSteps 20000
```

This case is the simplest intrinsically unsaturated triaxial example in the GCC mini:

- `PoreAirPressure 0.0` and `PoreWaterPressure -10.0` give an initial suction of `10 k Pa`
- that suction is then held constant during loading
- `VG_n`, `VG_m`, `VG_omega_prime`, `Pd`, `Pw`, `beta_x1`, and `beta_x2` control the retention and effective-stress update used during the path
- the driver still updates `S_w`, `\zeta`, `e`, and `SIGMC_unsat`, so this is not just a saturated triaxial run with a fixed suction label attached to it

Constant-p_net isotropic example Path: `mini_tools/GCCUMAT/cases/const_pnet/input.txt`

```
Mode ConstPnet
Phi 25.0
Lambda 0.11
Nu 0.30
Kappa 0.03
```

```
Alpha 0.77
Beta 1.0
Beta_prime 1.0
c1 0.2
c2 1.5
OCRControlled 0
DefaultIsoHardening 200
v_N 2.76
P_min 0.1
patm 100.0
STOL 1e-9
FTOL 1e-9
LTOL 1e-6
VG_n 2.5
VG_m 0.6
VG_omega_prime 0.2
Pd 0.05
Pw 0.1
b_w 5.0
b_d 5.0
b_sc 25.0
alpha_p_c 0.5
beta_x1 1.0
beta_x2 0.0
VoidRatio 1.5
StressXX -100
StressYY -100
StressZZ -100
PoreAirPressure 0.0
PoreWaterPressure -100.0
IsotropicHardening 200
OCR 2.0
Delta_vN 0.0
dSuction -4.5e-2
nSteps 2000
```

This packaged case starts from `100 kPa` suction and reduces it toward `10 kPa` while solving the isotropic volumetric response required to keep `p_net` constant. It is the clearest packaged GCC example for seeing that suction change alone can drive saturation, \zeta, and void-ratio evolution while the mechanical target remains isotropic.

Constant-water-content triaxial example Path: `mini_tools/GCCUMAT/cases/const_wcontent/input.txt`

```
Mode ConstWContent
Phi 25.0
Lambda 0.11
Nu 0.30
Kappa 0.03
Alpha 0.77
Beta 1.0
Beta_prime 1.0
c1 0.2
c2 1.5
OCRControlled 0
DefaultIsoHardening 200
v_N 2.76
P_min 0.1
patm 100.0
STOL 1e-9
FTOL 1e-9
LTOL 1e-6
VG_n 2.5
VG_m 0.6
VG_omega_prime 0.2
Pd 0.05
Pw 0.1
beta_x1 1.0
beta_x2 0.0
VoidRatio 1.5
StressXX -100
StressYY -100
StressZZ -100
PoreAirPressure 0.0
PoreWaterPressure -10.0
IsotropicHardening 200
OCR 2.0
Delta_vN 0.0
dEpsAxial -1e-5
nSteps 20000
```

This packaged case keeps water content fixed while loading triaxially, so suction, satura-tion, and void ratio evolve together through the nonlinear unsaturated constitutive update. Among the packaged GCC examples, this is the most strongly coupled path because the driver solves the mechanical target and the hydraulic water-content constraint together at the end of each step.

1.10.5 Output files and columns

Each run writes `stress_results.csv`.

Output file	Produced by	Main use
stress_results.csv	all cases	Main mechanical and hydraulic history used by the packaged GCC figures.

Primary output columns in stress_results.csv:

Output column	Meaning
step	Load-step index written by the standalone driver.
exx, eyy, ezz, ezy, ezx, exy	Strain components for the accepted step state.
sxx, syy, szz, szy, szx, sxy	Stress components for the accepted step state.
q, p, pnet	Deviatoric stress, mean stress, and net mean stress used by the GCC mini plots.
Sw, suction, e	Hydraulic state variables that are most useful for reading the bundled unsaturated examples.
zeta	GCC suction-dependent state variable coupling the hydraulic and mechanical updates.
IsoH	Saturated isotropic hardening variable stored by the GCC UMAT.

Derived unsaturated transformation outputs:

Output column	Meaning
a_zeta, b_zeta, c_zeta	Intermediate GCC transformation terms linking the saturated and unsaturated cap variables.
e_sat	Saturated reference void ratio at the current stress state.
SIGMC_unsat	Unsaturated cap size reported by the standalone driver.

When reading the packaged GCC outputs, a practical workflow is:

1. inspect q, p, and pnet first to understand the imposed loading path
2. inspect Sw, suction, e, and zeta next to see how the unsaturated state evolves

3. inspect `IsoH` and `SIGMC_unsat` if you want to follow mechanical hardening and the unsaturated cap size explicitly
4. use `a_zeta`, `b_zeta`, `c_zeta`, and `e_sat` only when you want to diagnose how the GCC transformation from saturated to unsaturated state is being assembled

The plots in the next section are generated from these packaged case CSVs.

1.11 Results

The plots below are produced directly from the bundled FALCON mini case inputs under `mini_tools/GCCUMAT/cases`. Together they show the saturated baseline behaviour and the intrinsically unsaturated GCC responses under three different hydraulic constraints.

1.11.1 Saturated drained triaxial
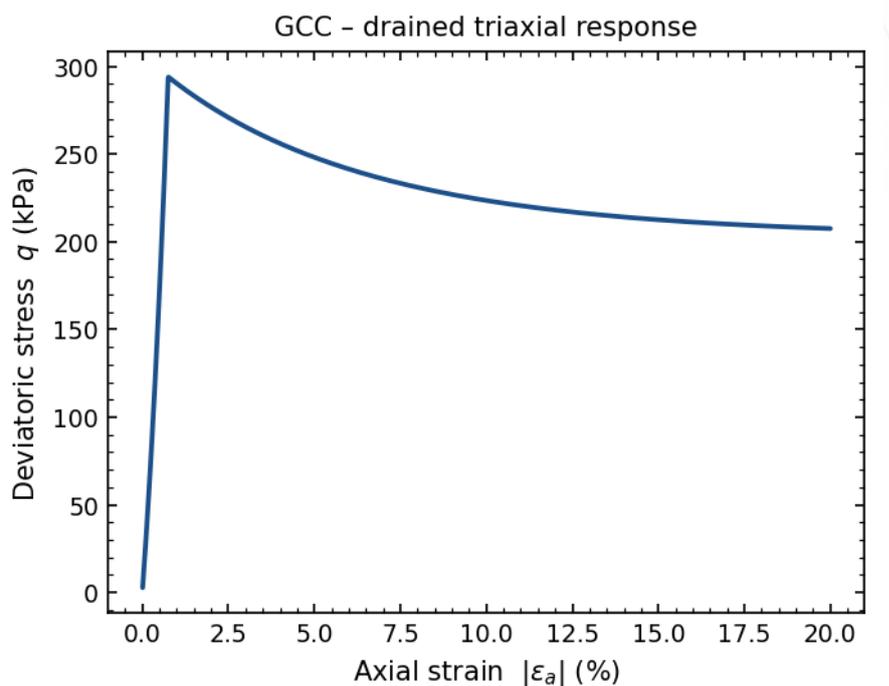
Bundled case `cases/drained/input.txt`. This is the saturated drained baseline used for the GCC mini. The figure is intentionally simple: it gives the reference $q-\varepsilon_a$ response before any suction coupling or retention-law effects are introduced.

1.11.2 Saturated undrained triaxial

Bundled case `cases/undrained/input.txt`. This packaged input is included so the same GCC parameter set can be exercised under a saturated constant-volume branch. In the current

documentation, the main emphasis remains on the unsaturated paths because that is where the GCC mini differs most strongly from the simpler saturated models.

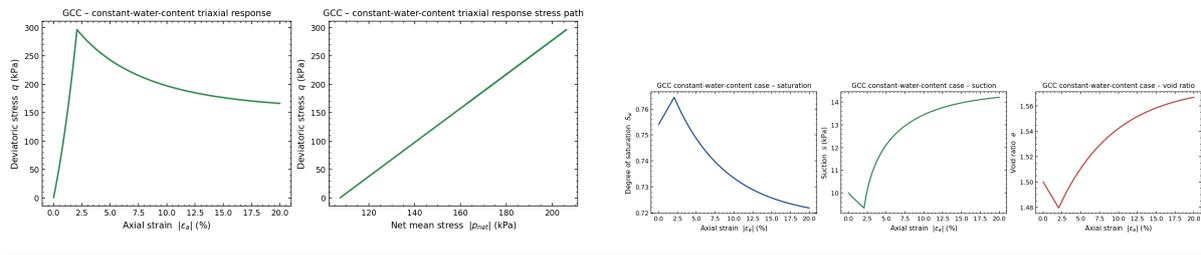### 1.11.3 Unsaturated constant-suction triaxial
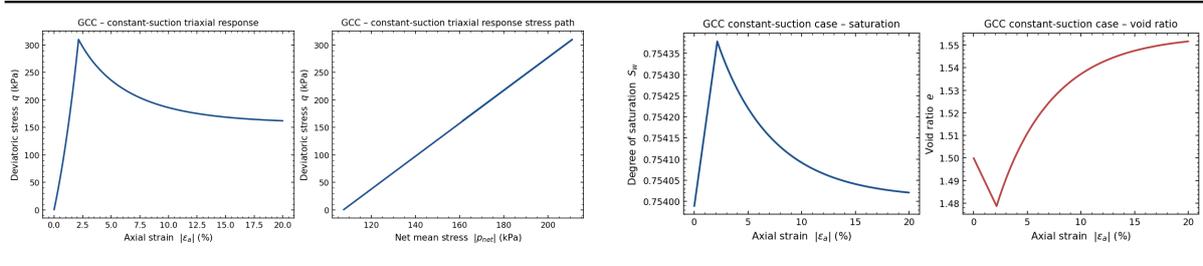
Bundled case `cases/qoverpnet/input.txt`. Left: mechanical response in axial-strain and `q-p_net` space. Right: saturation and void-ratio evolution for the constant-suction packaged case. This example is useful when you want to isolate the effect of unsaturated effective stress and GCC state-variable coupling without also changing the imposed suction during the run.

1.11.4 Unsaturated constant-p_net isotropic loading
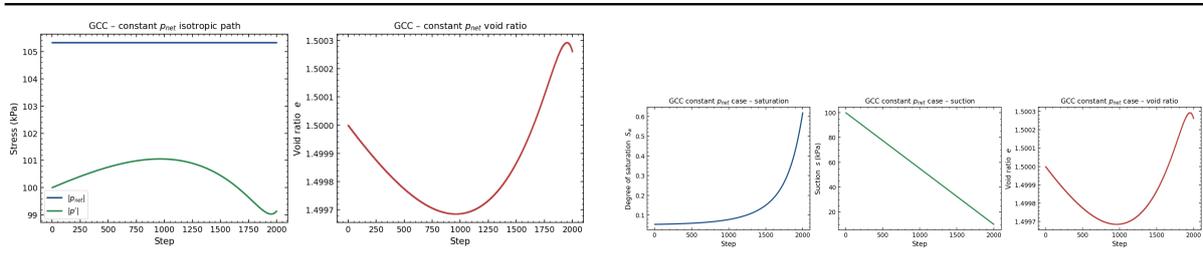
Bundled case `cases/const_pnet/input.txt`. Left: stress and void-ratio evolution for the constant-`p_net` isotropic path. Right: saturation, suction, and void-ratio evolution from the same packaged case. This is the clearest packaged GCC example for seeing suction-driven hydraulic changes while the mechanical target is an isotropic `dp_net = 0` path, because q remains zero and the figure is not obscured by deviatoric plasticity.

1.11.5 Unsaturated constant-water-content triaxial

Bundled case `cases/const_wcontent/input.txt`. Left: mechanical response in axial-strain and `q-p_net` space. Right: saturation, suction, and void-ratio evolution from the packaged constant-water-content example. This is the most useful packaged GCC case for understanding the fully coupled flavour of the model: water content is fixed, but suction, degree of saturation, void ratio, and hardening still evolve through the constitutive update.

1.12 Applications and limitations

- Best suited to fine-grained soils and unsaturated critical-state problems where suction-dependent hardening is required inside the constitutive law itself.
- Appropriate for uncoupled, coupled, and fully coupled analyses when combined with consistent phase, permeability, SWRC, and effective-stress definitions.
- Not intended as a liquefaction-oriented cyclic sand model; for that class of problem, `sanisand_type`, NorSand, or Multi-Yield-type models are more appropriate.

1.13 References

- **Borja (2004)**: Borja, R. I. (2004). *Cam-Clay plasticity. Part V: A mathematical framework for three-phase deformation and strain localization analyses of partially saturated porous media.* Computer Methods in Applied Mechanics and Engineering, 193(48-51), 5301–5338.

- **Gallipoli et al. (2003)**: Gallipoli, D., Gens, A., Sharma, R., & Vaunat, J. (2003). *An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour.* Géotechnique, 53(1), 123–135.

- **Ghorbani et al. (2018)**: Ghorbani, J., Airey, D. W., & El-Zein, A. (2018). *Numerical framework for considering the dependency of SWCCs on volume changes and their hysteretic responses in modelling elasto-plastic response of unsaturated soils.* Computer Methods in Applied Mechanics and Engineering, 336, 80–110.

- **Ghorbani & Kodikara (2024)**: Ghorbani, J., & Kodikara, J. (2024). *Thermodynamically consistent effective stress formulation for unsaturated soils across a wide range of soil saturation.* Computational Mechanics, 73(5), 1077–1094.

- **Perić (2006)**: Perić, D. (2006). *Analytical solutions for a three-invariant Cam clay model subjected to drained loading histories.* International Journal for Numerical and Analytical Methods in Geomechanics, 30, 363–387.

- **Sheng et al. (2000)**: Sheng, D., Sloan, S. W., & Yu, H. S. (2000). *Aspects of finite element implementation of critical state models.* Computational Mechanics, 26(2), 185–196.