AD FALCON API Manual

Multi-Yield Model (Yang, Elgamal, and Parra, 2003)

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1 Multi-Yield Model (Yang, Elgamal, and Parra, 2003)

The `ElgamalMultiYieldUMAT` implements the Yang, Elgamal, and Parra (2003) cyclic-mobility formulation. The model uses a Prevost-style pressure-dependent multi-yield framework with a strain-space translation/enlargement mechanism for large cyclic shear deformation.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/ElgamalMultiYieldUMAT.cpp path/to/ElgamalMultiYieldUMAT.hpp
Mechanical \
    Gr_kPa=33300 phi_deg=31.4 phiPT_deg=26.5 c1=0.075 c2=1000 \
    d1=200 d2=1.5 py_kPa=10 gammaSMax=0.015 R=1 \
    po_kPa=1 pr_kPa=80 n=0.5 nu=0.30 gammaF=0.10 NYS=18 \
    enable_translation=1 gamma_d_total=1 \
    p_regularization_eps=1e-3 p_zero_snap=1e-8
```

For readability, this example is wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

Use the parameter names listed below.

1.2 Material parameters

Symbol	Keyword in input	Units	Required	Description
G_r	`Gr_kPa`	stress	✓	Reference shear modulus at p'_r.
ϕ_f	`phi_deg`	°	✓	Friction angle controlling the failure stress ratio.
ϕ_{PT}	`phiPT_deg`	°	✓	Phase-transformation friction angle.

Symbol	Keyword in input	Units	Required	Description
c_1	`c1`	–	✓	Contraction coefficient in the pre-phase-transformation branch.
c_2	`c2`	–	✓	Contraction hardening coefficient acting through ε_c.
d_1	`d1`	–	✓	Dilation coefficient in the post-phase-transformation branch.
d_2	`d2`	–	✓	Exponent on accumulated dilative shear strain.
p_y	`py_kPa`	stress	✓	Pressure threshold governing the strain-space domain size γ_s.
$\gamma_{s,\max}$	`gammaSMax`	strain	✓	Maximum strain-space domain size.
R	`R`	–	✓	Cap on the translation distance of the strain-space domain.
p_0'	`po_kPa`	stress	✓	Positive pressure shift used in confinement scaling and stress-ratio evaluation.
p_r'	`pr_kPa`	stress	✓	Reference confinement in the modulus scaling law.
n	`n`	–	✓	Confinement-scaling exponent.

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Symbol	Keyword in input	Units	Required	Description
ν	`nu`	–	✓	Poisson ratio used with the pressure-dependent elastic bulk modulus.
γ_f	`gammaF`	strain	✓	Backbone end strain at the reference confinement.
`NYS`	`NYS`	–	✓	Number of nested yield surfaces used in the multi-yield discretization.
`enable_translation`	`enable_translation`	–	×	Enables the strain-space translation rule (1) or keeps the center fixed (0).
`gamma_d_total`	`gamma_d_total`	–	×	Uses total deviatoric strain (1) or plastic deviatoric strain (0) in the dilative branch accumulator γ_d.
ε_p^{reg}	`p_regularization_eps`	stress	×	Smoothing width used to regularize p' near zero in modulus, strength, and plastic-flow evaluations.
p'_{snap}	`p_zero_snap`	stress	×	Threshold below which the committed mean effective stress is snapped to exactly zero after the constitutive update.

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1.3 Custom state variables

The UMAT maintains a structured family of history variables under the `ELG_*` prefix.

Per-surface deviatoric stresses:

- `ELG_s0_xx` … `ELG_s0_xy`
- `ELG_s1_xx` … `ELG_s1_xy`
- …
- `ELG_s(NYS-2)_xx` … `ELG_s(NYS-2)_xy`

Strain-space history:

- `ELG_epsdev_xx`, `ELG_epsdev_yy`, `ELG_epsdev_zz`, `ELG_epsdev_yz`, `ELG_epsdev_zx`, `ELG_epsdev_xy`
- `ELG_epscenter_xx`, `ELG_epscenter_yy`, `ELG_epscenter_zz`, `ELG_epscenter_yz`, `ELG_epscenter_zx`, `ELG_epscenter_xy`
- `ELG_epsunload_xx`, `ELG_epsunload_yy`, `ELG_epsunload_zz`, `ELG_epsunload_yz`, `ELG_epsunload_zx`, `ELG_epsunload_xy`

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Scalar history / diagnostics:

- `ELG_eta_prev`
- `ELG_eps_c`
- `ELG_gamma_d`
- `ELG_gamma_d_max`
- `ELG_tau_bias_lp`
- `ELG_gamma_unload`
- `ELG_was_unloading`
- `ELG_phase`
- `ELG_active`

Optional diagnostic outputs exposed by the standalone driver include:

- `ELG_p_eff`, `ELG_p_trial`, `ELG_p_unclamped`, `ELG_p_clamped`
- `ELG_tau`
- `ELG_gamma_p`
- `ELG_dLambda`
- `ELG_Pdd`
- `ELG_dEvP`
- `ELG_gamma_s`
- `ELG_gamma_dist`
- `ELG_center_mag`
- `ELG_unloading_shear`
- `ELG_eta_dot_sign`
- `ELG_outside_pt`
- `ELG_unloading_eta`

- `ELG_neutral_now`
- `ELG_substeps`
- `ELG_relerr_max`

1.4 Backbone and confinement scaling

At the reference confinement, the model uses a hyperbolic octahedral backbone:

$$\tau = \frac{G_r \gamma}{1 + \gamma/\gamma_r} \tag{1}$$

where γ_r is chosen so that $\tau(\gamma_f) = \tau_f$ at $p' = p'_r$.

The failure and phase-transformation stress ratios are

$$M_f = \frac{6 \sin \phi_f}{3 - \sin \phi_f}, \qquad M_{PT} = \frac{6 \sin \phi_{PT}}{3 - \sin \phi_{PT}} \tag{2}$$

and the corresponding octahedral shear strengths are

$$\tau_f = \frac{\sqrt{2}}{3} M_f \left(p' + p'_0\right), \qquad \tau_{PT} = \frac{\sqrt{2}}{3} M_{PT} \left(p' + p'_0\right). \tag{3}$$

The confinement scaling used by the code is

$$s_p = \frac{p' + p'_0}{p'_r + p'_0}, \qquad G(p') = G_r\, s_p^n \tag{4}$$

with bulk modulus

$$K(p') = \frac{2G(p')(1 + \nu)}{3 - 6\nu}. \tag{5}$$

The hyperbolic backbone is discretized into `NYS` nested surfaces, and the implementation uses an Iwan/Prevost-style parallel-spring update to recover the multisurface response.

At the reference confinement, the paper defines the piecewise-linear multisurface parameters as

$$H_m = 2\,\frac{\tau_{m+1} - \tau_m}{\gamma_{m+1} - \gamma_m}, \qquad m = 1, \ldots, \text{NYS}, \tag{3a}$$

with

$$H_{\text{NYS}} = 0, \tag{3b}$$

and the size of the `m`th surface as

$$M_m = \frac{3\tau_m}{\sqrt{2}\,(p'_r + p'_0)}, \qquad M_{\text{NYS}} = M_f. \tag{4a}$$

These are the `H_m` and `M_m` quantities referred to in Yang, Elgamal, and Parra (2003). In the present implementation they are built internally from the chosen backbone and the user-specified `NYS`, rather than being entered one by one in the input file.

1.5 Yield family and stress update

The paper's yield family is a set of similar conical surfaces with common apex at `-p'_0` on the hydrostatic axis. In effective-stress form, the `mth` yield function may be written as

$$f_m(\boldsymbol{\sigma}', \boldsymbol{\alpha}) = \frac{3}{2}(\boldsymbol{s} - p'\boldsymbol{\alpha}) : (\boldsymbol{s} - p'\boldsymbol{\alpha}) - M_m^2\,(p' + p'_0)^2 = 0, \tag{6a}$$

where

$$\boldsymbol{s} = \boldsymbol{\sigma}' - p'\boldsymbol{I}. \tag{6b}$$

Here `M_m` is the size of the `mth` conical surface and `\boldsymbol{\alpha}` is the deviatoric translation tensor that shifts the surface center in stress space. The outermost surface corresponds to the failure surface through `M_f`.

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For each surface m, the octahedral shear strength τ_m is scaled with confinement, and the corresponding deviatoric radius in tensor norm is

$$r_m = \sqrt{3}\,\tau_m(p'). \tag{6}$$

The deviatoric trial stress of each spring is projected back to its active radius if the trial norm exceeds r_m. Summing the spring contributions gives the updated deviatoric stress $\boldsymbol{s}_{n+1}$.

The mean stress update is written as

$$p'_{trial} = p'_n + K\,\Delta\varepsilon_v \tag{7}$$

followed by a plastic-volume correction

$$p'_{n+1} = p'_{trial} - K\,\Delta\varepsilon_v^p. \tag{8}$$

The code applies this correction with a small fixed-point iteration because K itself depends on the updated mean stress.

1.6 Volumetric flow and cyclic mobility mechanism

The model uses octahedral quantities and a scalar volumetric-flow measure P''. Let

$$\eta = \frac{q}{p' + p'_0} \tag{9}$$

and define the strain-space domain size

$$\gamma_s = \gamma_{s,\max} \left\langle \frac{p_y - p'}{p_y} \right\rangle, \tag{10}$$

where $\langle \cdot \rangle$ denotes the Macaulay bracket.

The code tracks the current deviatoric strain position ε_{dev} and the domain center ε_{center}, with distance

$$\gamma_{dist} = 2\sqrt{\frac{1}{3}(\varepsilon_{dev} - \varepsilon_{center}) : (\varepsilon_{dev} - \varepsilon_{center})}. \tag{11}$$

The implemented volumetric-flow law is piecewise:

$$P'' = \begin{cases} \left(1 - \operatorname{sign}(\dot{\eta})\dfrac{\eta}{\eta_{PT}}\right)(c_1 + c_2\varepsilon_c), & \text{below PT or during unloading,} \\ 0, & \text{inside the neutral strain-space domain,} \\ \left(1 - \dfrac{\eta}{\eta_{PT}}\right) d_1 \gamma_d^{d_2}, & \text{outside PT and outside the neutral domain.} \end{cases} \tag{12}$$

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The plastic volumetric strain increment is then

$$\Delta\varepsilon_v^p = 3P''\Delta\lambda, \qquad \Delta\lambda = \Delta\gamma_p, \tag{13}$$

where $\Delta\gamma_p$ is the octahedral plastic shear increment recovered from the spring return.

The contraction measure ε_c is updated by accumulated contractive plastic volume, while the dilative measure γ_d is accumulated after phase transformation. This is the mechanism that reproduces the rapid strain accumulation associated with cyclic mobility.

1.7 Translation rule

Under biased cyclic loading, the yield domain is translated in deviatoric strain space. In the present implementation, when `enable_translation=1`, the translation increment is written as

$$\gamma_r = \min(R\,\gamma_s,\ R\,\gamma_{\text{unload}}), \tag{14}$$

where `\gamma_s` is the current strain-space radius from Eq. (10) and `\gamma_{\text{unload}}` is the accumulated unloading strain. The center shift is then applied in the normalized unloading-strain direction:

$$\Delta\varepsilon_{center} = \gamma_r \frac{\varepsilon_{unload}}{\|\varepsilon_{unload}\|}. \tag{14a}$$

In the packaged mini cases:

- symmetric cyclic simple shear uses `enable_translation=0`
- biased cyclic simple shear uses `enable_translation=1`
- `R=1` and `R=3` are provided as separate examples

1.8 Low mean-stress regularization

The implementation is explicitly regularized as $p' \to 0$:

$$\bar{p}' = \frac{1}{2}\left(p' + \sqrt{(p')^2 + \varepsilon_{reg}^2}\right), \qquad \varepsilon_{reg} = \texttt{p\_regularization\_eps}. \tag{15}$$

The regularized mean stress $\bar{p}'$ is used in the confinement scaling, elastic moduli, phase-transformation checks, and strength calculations, so modulus and stress-ratio terms remain bounded when the stress path reaches the liquefaction floor. After the constitutive correction, any committed mean stress smaller than `p_zero_snap` is written back as exactly zero for restart/output consistency.

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The packaged mini includes a low-confinement q-controlled probe that exercises the regularization to its floor.

1.9 FALCON mini

The packaged mini tool id is `Elgamal`. It lives under `mini_tools/Elgamal`.

1.9.1 How to run

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool Elgamal
--mini-input
/path/to/UMATLIB_FALCON/falcon_minis/Elgamal/cases/triaxial_cidc_p80
```

Packaged simulation families:

Packaged case	Path	Purpose
Monotonic simple shear	`cases/monotonic_simple_shear_p10/input.txt`	Monotonic simple-shear backbone at low confinement.
Cyclic simple shear, symmetric	`cases/cyclic_simple_shear_sym_r1/input.txt`	Symmetric cyclic simple shear with translation disabled.

Packaged case	Path	Purpose
Cyclic simple shear, biased `R = 1`	`cases/cyclic_simple_shear_bias_r1/input.txt`	Biased cyclic simple shear with modest translation.
Cyclic simple shear, biased `R = 3`	`cases/cyclic_simple_shear_bias_r3/input.txt`	Biased cyclic simple shear with stronger translation effect.
Drained triaxial	`cases/triaxial_cidc_p80/input.txt`	Drained triaxial reference path.
Cyclic triaxial `q`-controlled	`cases/cyclic_triaxial_q/input.txt`	Undrained cyclic triaxial stress-controlled example.

1.9.2 Input syntax

`input.txt` uses whitespace-delimited `Key Value` pairs, one item per line, for example:

```
Mode triaxial_cidc
OutputCSV triaxial_cidc_results.csv
InitialMeanStressKPa 80.0
GammaMax 0.08
```

The main selector is `Mode`.

`Mode` value	Meaning in the standalone mini
`monotonic_simple_shear`	Monotonic simple shear at prescribed initial mean effective stress.
`cyclic_simple_shear`	Cyclic simple shear driven by shear-strain increments; the loading direction reverses when the target shear-stress bound is reached or the response is capped by the low-`p'` state.
`triaxial_cidc`	Drained triaxial loading at approximately constant radial effective stress.
`cyclic_triaxial_q`	Undrained cyclic triaxial loading controlled by deviatoric stress.

Mini inputs used by the packaged cases:
Driver controls:

Input key	Used by	Required / choices / defaults	Meaning
`Mode`	all cases	Required; choices listed in the mode table above	Selects the loading program.
`OutputCSV`	all cases	Optional; case-specific default if omitted	Output CSV name written in the case directory.
`InitialMeanStressKPa`	all cases	Required in packaged cases	Starting mean effective stress.
`nSteps`	monotonic and drained triaxial	Required for monotonic / drained triaxial modes	Number of driver steps.
`nCycles`, `stepsPerCycle`	cyclic cases	Required for cyclic modes	Number of cycles and number of steps per cycle.
`GammaMax`	simple shear and drained triaxial	Required in the packaged backbone cases	Shear/axial strain target magnitude used by the standalone driver.
`TauAmpKPa`, `TauBiasKPa`	cyclic simple shear	Required for cyclic simple-shear modes	Upper/lower shear-stress bounds used by the strain-driven reversal logic. The driver keeps straining in one direction until it reaches the bound, then reverses.
`QBiasKPa`, `QAmpKPa`	cyclic triaxial `q` control	Required for `Mode = cyclic_triaxial_q`	Mean and amplitude of the target deviatoric-stress history.
`PhaseShiftCycles`, `ControlTauOct`, `InputTauIsOct`, `PreloadInitial`, `MaxSplitDepth`	selected cyclic cases	Optional; driver defaults if omitted	Advanced driver controls for the cyclic simple-shear and triaxial branches.

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Constitutive inputs:

Input key	Used by	Required / choices / defaults	Meaning
`Gr_kPa`, `nu`, `pr_kPa`, `n`	all cases	Required in packaged cases	Elastic reference stiffness and pressure dependence parameters.
`phi_deg`, `phiPT_deg`, `py_kPa`	all cases	Required in packaged cases	Yield and phase-transformation stress-ratio controls.
`c1`, `c2`, `d1`, `d2`	all cases	Required in packaged cases	Multi-surface evolution and contraction/dilation parameters.
`gammaSMax`, `gammaF`, `gamma_d_total`	all cases	Required in packaged cases	Shear-strain scale parameters.
`R`, `enable_translation`	cyclic cases	Required in packaged cyclic cases; `enable_translation` choices `0` / `1`	Translation strength and translation on/off switch for cyclic mobility response.
`p0_kPa`, `NYS`	all cases	Required in packaged cases	Yield-surface distribution controls.
`p_regularization_eps`, `p_zero_snap`	all cases	Optional but recommended; driver defaults if omitted	Low-`p'` regularization controls in the standalone mini and UMAT.

1.9.3 Hydromechanical assumptions

The packaged Multi-Yield mini is an effective-stress mechanical driver:

- there is no unsaturated retention or suction update
- all cyclic mobility behaviour comes from the multi-surface constitutive law itself
- low-`p'` behaviour is regularized numerically through `p_regularization_eps` and `p_zero_snap`

This mini is therefore best read as a cyclic effective-stress sand driver, not as a general-purpose unsaturated or monotonic critical-state model.

1.9.4 Sample input

Drained triaxial example Path: `mini_tools/Elgamal/cases/triaxial_cidc_p80/input.txt`

```
Mode triaxial_cidc
OutputCSV triaxial_cidc_results.csv
InitialMeanStressKPa 80.0
GammaMax 0.08
nSteps 500
R 1.0
enable_translation 1
```

This is the packaged drained triaxial reference path. It gives the simplest axisymmetric view of the multi-yield model without the additional complexity of cyclic reversal.

Monotonic simple-shear example Path: `mini_tools/Elgamal/cases/monotonic_simple_shear_p10/input.txt`

This packaged case is the cleanest way to inspect the simple-shear backbone and the evolution of shear stress at low confinement.

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Cyclic simple-shear examples Paths:

- `mini_tools/Elgamal/cases/cyclic_simple_shear_sym_r1/input.txt`
- `mini_tools/Elgamal/cases/cyclic_simple_shear_bias_r1/input.txt`
- `mini_tools/Elgamal/cases/cyclic_simple_shear_bias_r3/input.txt`

These packaged cases are used to compare symmetric and biased simple-shear loading, and to show how the translation parameter `R` changes the cyclic response. In all three cases the mini applies shear-strain increments and reverses the strain direction when the target shear-stress level is reached, or when the low-`p'` state prevents further stress build-up in that direction.

Cyclic triaxial q-controlled example Path: `mini_tools/Elgamal/cases/cyclic_triaxial_q/input.txt`

This packaged case is the triaxial companion to the simple-shear cyclic examples. It is useful when you want to study cyclic effective-stress reduction in an axisymmetric loading program rather than in simple shear.

1.9.5 Output files and columns

Each packaged case writes its own mode-specific CSV such as `triaxial_cidc_p80_results.csv`, `monotonic_simple_shear_results.csv`, or `cyclic_triaxial_q_results.csv`.

Output family	Main columns
Simple shear	`gamma_xy`, `gamma_oct`, `tau_xy`, `tau_oct`, `p_eff`, `eta`
Triaxial	`eps_a`, `eps_r`, `q_triax`, `tau_oct`, `p_eff`, `eta`
Multi-surface state	`Pdd`, `gamma_p`, `dLambda`, `dEvP`, `eps_c`, `phase`, `active`
Translation / reversal state	`gamma_s`, `gamma_dist`, `center_mag`, `tau_bias_lp`, `gamma_unload`
Numerical diagnostics	`substeps`, `relerr_max`, `p_clamped`, and related flags

The standalone driver accepts either `./input.txt`, an explicit input-file path, or a case directory. The plots in the next section are generated from these packaged case CSVs.

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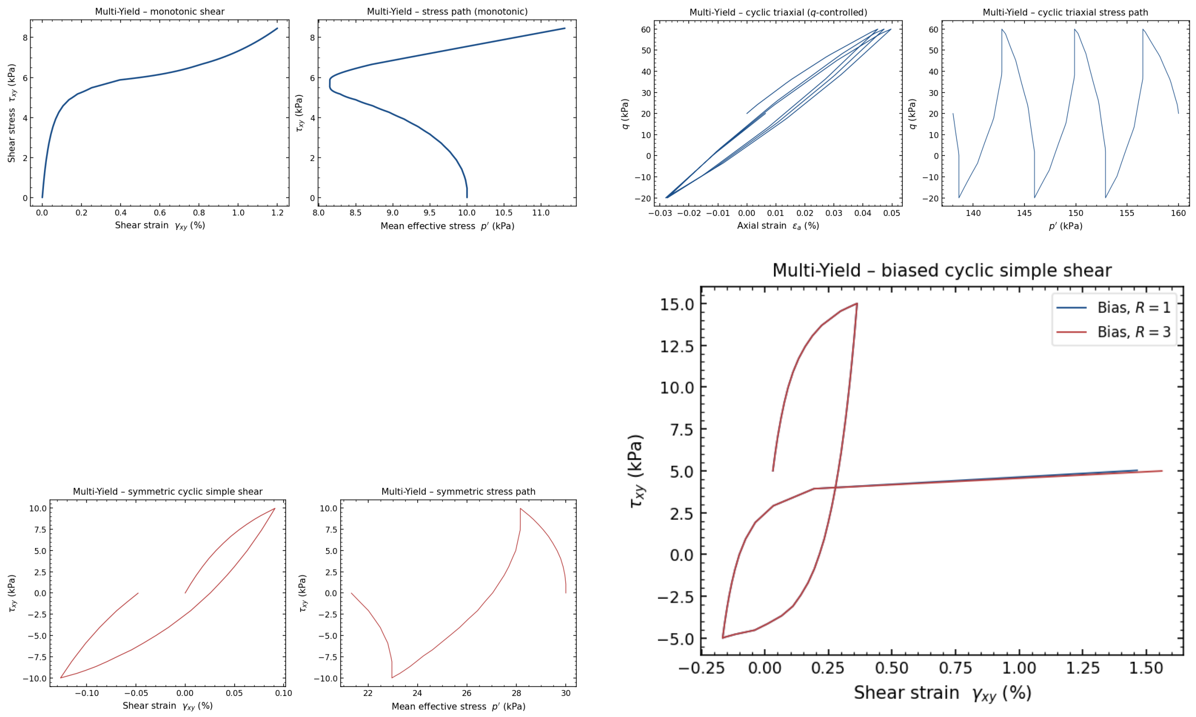
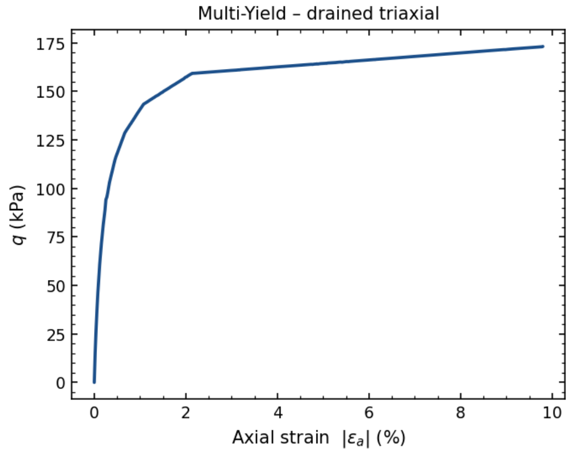
1.10 Results

The plots below are produced directly from the bundled FALCON mini case inputs under `mini_tools/Elgamal/cases`. The packaged examples use the same multi-yield parameter family with case-specific loading programs.





Top-left uses `cases/monotonic_simple_shear_p10/input.txt`. Top-right uses `cases/cyclic_triaxial_q/input.txt`. Middle-left uses `cases/cyclic_simple_shear_sym_r1/input.txt`. Middle-right compares `cases/cyclic_simple_shear_bias_r1/input.txt` and `cases/cyclic_simple_shear_bias_r3/input.txt`. Bottom-left uses `cases/triaxial_cidc_p80/input.txt`.

1.10.1 Selected paper reproductions

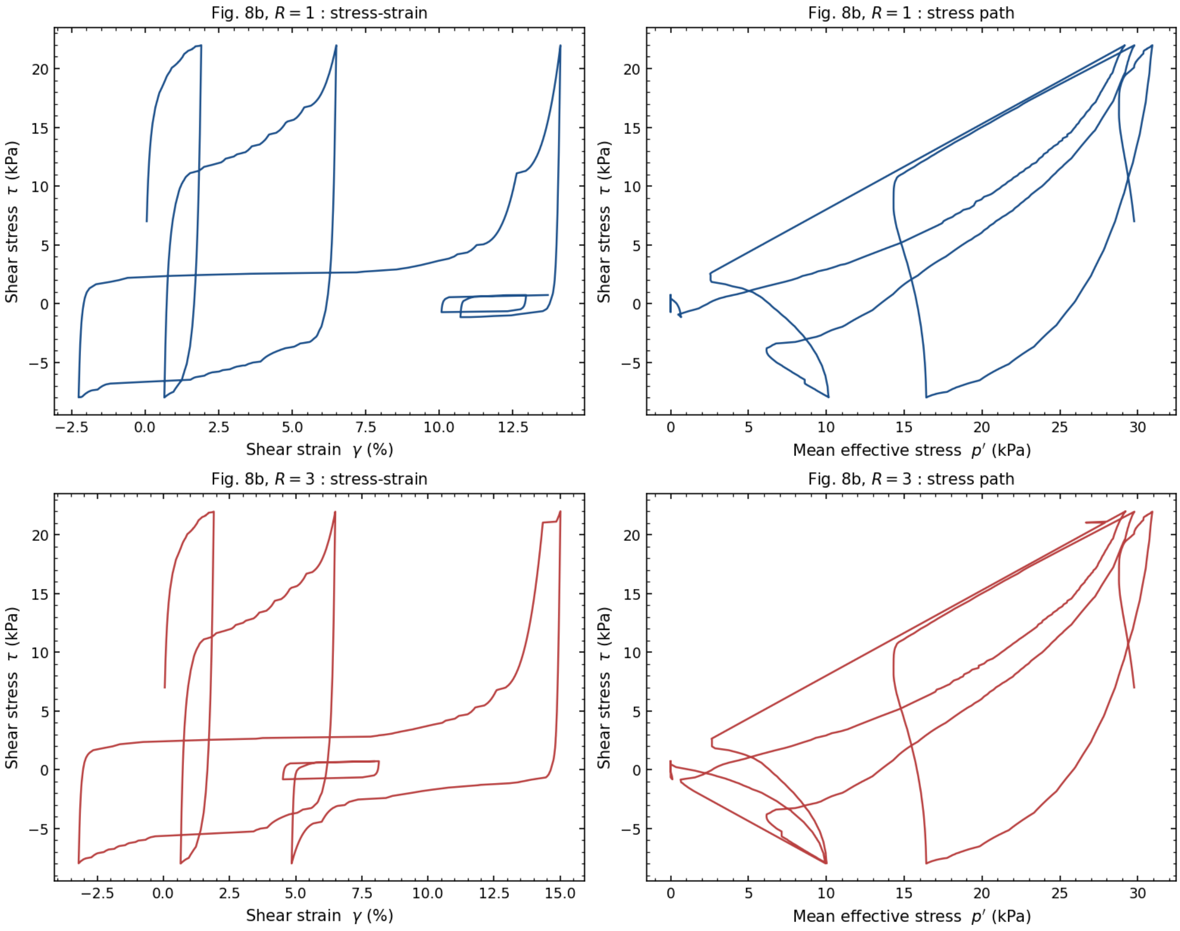


Fig. 8b-style stress-controlled undrained cyclic simple-shear reproduction from the packaged mini cases `cases/paper_fig8_bias_r1/input.txt` and `cases/paper_fig8_bias_r3/input.txt`. The plot uses the regenerated `R=1` and `R=3` case outputs directly, with ± 15 kPa cyclic amplitude and a 7 kPa static shear-stress bias.

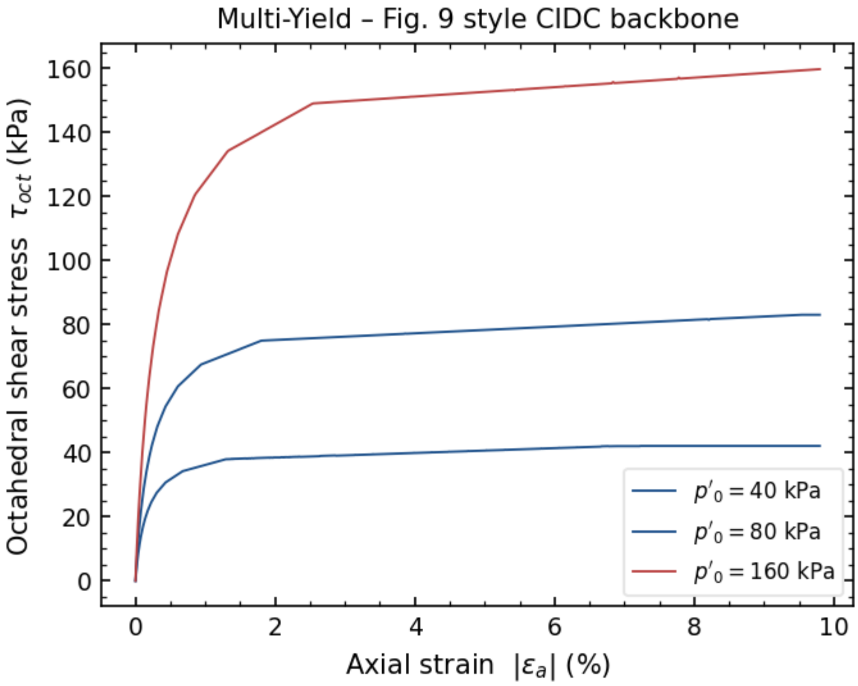


Fig. 9-style drained CIDC backbone reproduction using `cases/paper_fig9_cidc_p40/input.txt`, `cases/paper_fig9_cidc_p80/input.txt`, and `cases/paper_fig9_cidc_p160/input.txt`.

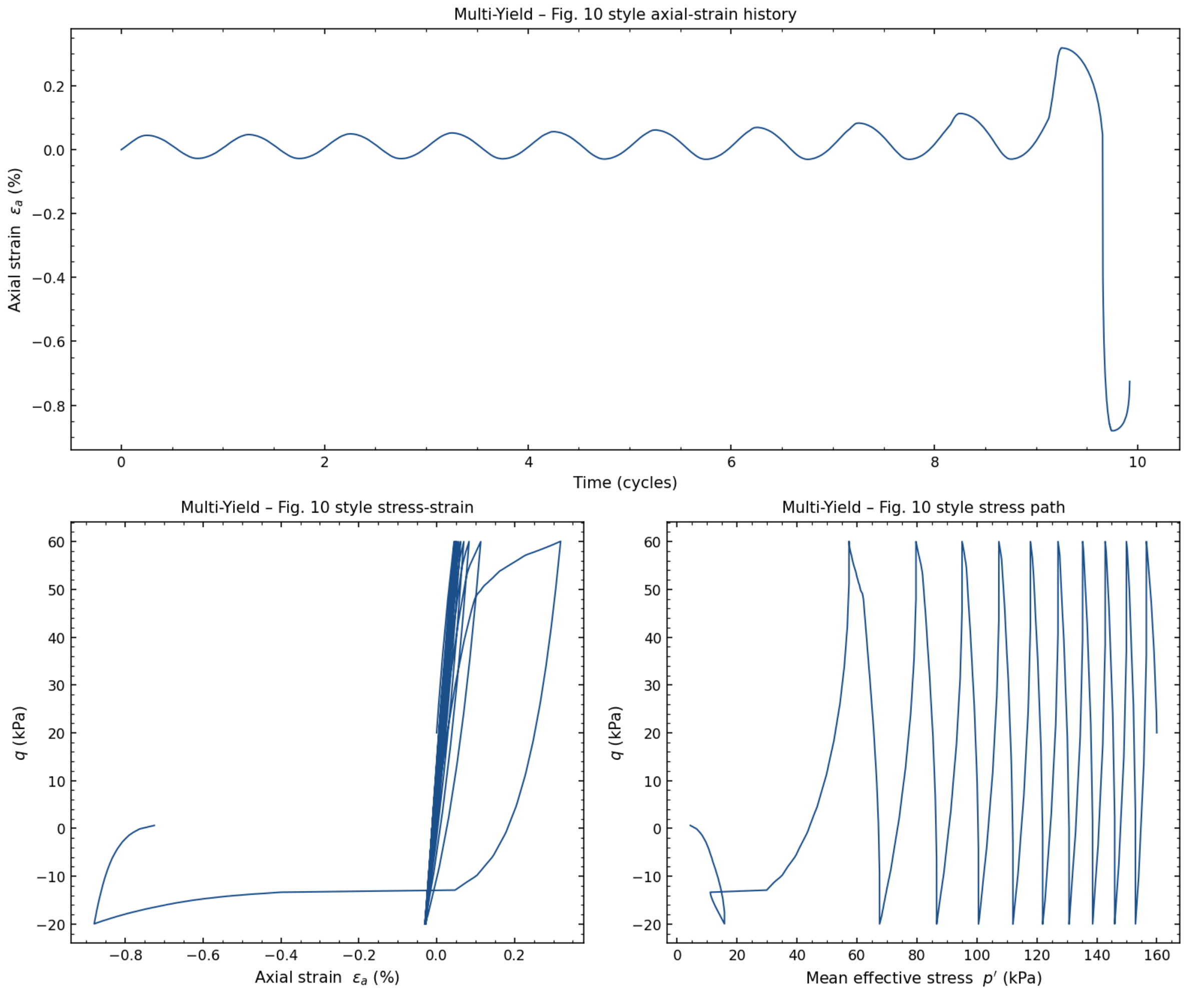


Fig. 10-style cyclic triaxial reproduction using `cases/paper_fig10_cyclic_triaxial/input.txt`. The figure shows axial-strain history, q versus axial strain, and the q-p' stress path from the packaged paper-style case.

1.11 Applications and limitations

- Best suited to cyclic mobility and liquefaction-oriented effective-stress studies on medium-dense sands, especially under simple-shear and cyclic triaxial loading paths.
- Appropriate for uncoupled and effective-stress-based coupled analyses where the main interest is cyclic strain accumulation rather than suction-dependent constitutive coupling.
- This is not a general unsaturated constitutive law and is not intended for clay critical-state behavior, rock-mass strength, or broad-purpose monotonic soil modeling outside its calibrated range.

1.12 References

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