AD FALCON API Manual

Elastic Contact Problems and the Penalty Coefficient Effect

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1 Elastic Contact Problems and the Penalty Coefficient Effect

1.1 Example 1: Footing on Soil (Non-Frictional)

1.1.1 Input File Name

fem_footing_conforming.txt

Input files used to generate Figures 2–3 (penalty sweep + AL comparison): - fem_footing_conforming_penalty_kn1e5.txt - fem_footing_conforming_penalty_kn5e5.txt - fem_footing_conforming_penalty_kn1e6.txt - fem_footing_conforming_penalty_kn5e6.txt - fem_footing_conforming_penalty_kn1e7.txt - fem_footing_conforming_AL_kn1e6.txt

1.1.2 Problem Description

A smooth, rigid strip footing of half-width $a = 0.15$ m (full width $B = 0.3$ m) rests on an elastic half-space. Contact is modeled via a frictionless penalty formulation using the mortar approach.

The rigid object was initially positioned 1 mm above the surface and then moved downward to establish contact with the soil, simulating a controlled displacement-driven contact scenario.

1.1.3 Model Setup

- **Footing**: Smooth rigid strip

- **Half-width**: $a = 0.15$ m
- **Soil**: Linear elastic, $E = 100$ MPa, $\nu = 0.3$
- **Contact**:
  - Normal penalty: k_n (varied—see below)
  - Friction coefficient: $\mu = 0$

- **Loading**: Uniform downward displacement $\delta = 2$ mm applied to the footing.

1.1.4 Analytical Solution

For plane-strain contact of a rigid strip on an elastic half-space, the centerline pressure is

$$p(0) = \frac{E\,\delta}{\pi\,(1-\nu^2)\,a} = \frac{100\times10^6 \times 1\times10^{-3}}{\pi\,(1-0.3^2)\,0.15} \approx 2.33\times10^5\ \text{Pa} = 233\ \text{kPa} \tag{1}$$

Penalty-Parameter Study A characteristic feature of the penalty formulation is that the accuracy of the contact enforcement depends on the magnitude of the penalty coefficient k_n. In the ideal limit $k_n \to \infty$, the numerical solution converges to the exact frictionless constraint,

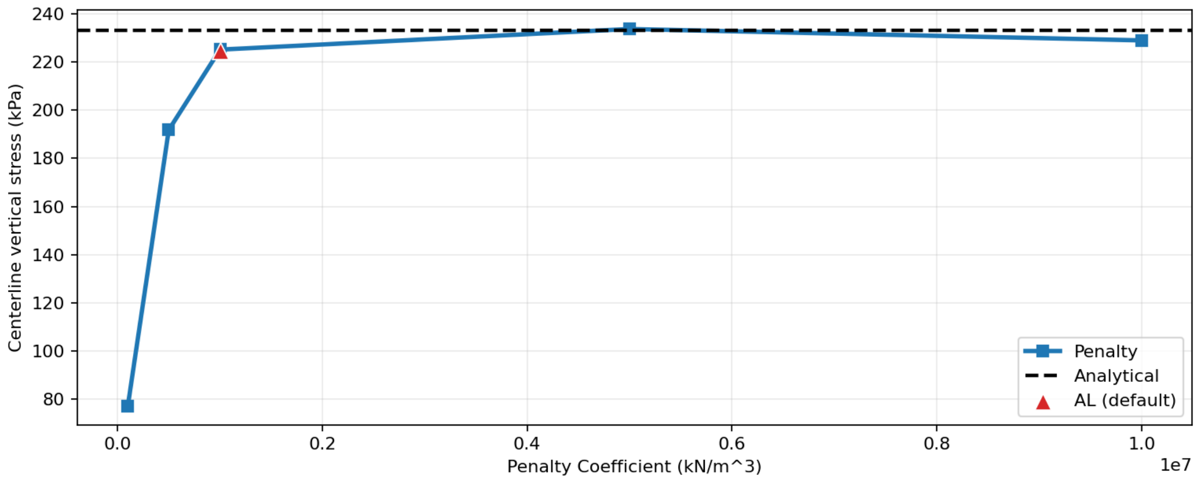

Figure 1: Vertical stress beneath the center vs penalty coefficient

yielding perfect agreement with the analytical result. However, in practice, excessively large k_n values introduce numerical stiffness, which can manifest as oscillations, slow convergence, or even premature solver failure.

Therefore, k_n must be chosen large enough to ensure acceptable accuracy, yet not so large that it impairs solver robustness. There is no universal guideline for this choice; the optimal penalty is problem-specific and often determined by numerical experimentation.

A series of simulations was carried out by varying k_n, recording the **centerline** vertical stress σ_{yy} at a probe located slightly below the soil surface and the total analysis time. As k_n increases, the predicted pressure approaches the analytical value $p_{\max} = 233\ \mathrm{kPa}$, but the computational time also grows.

In addition, one simulation was rerun using the **Augmented Lagrangian (AL)** normal-contact formulation (default AL parameters) at $k_n = 10^6$ to illustrate how AL compares against pure penalty enforcement for a representative penalty level.

1.1.5 Figures

Figure 2: Vertical stress beneath the center of the footing as a function of penalty coefficient (penalty formulation), with one Augmented Lagrangian (AL) run shown for comparison.

Figure 3: Analysis time as a function of penalty coefficient (penalty formulation), with one Augmented Lagrangian (AL) run shown for comparison.

2 Example 2: Rigid Cylinder on Soil (Plane-Strain, Frictionless)

2.0.1 Input File Name

fem_cylinder_soil_interaction.txt

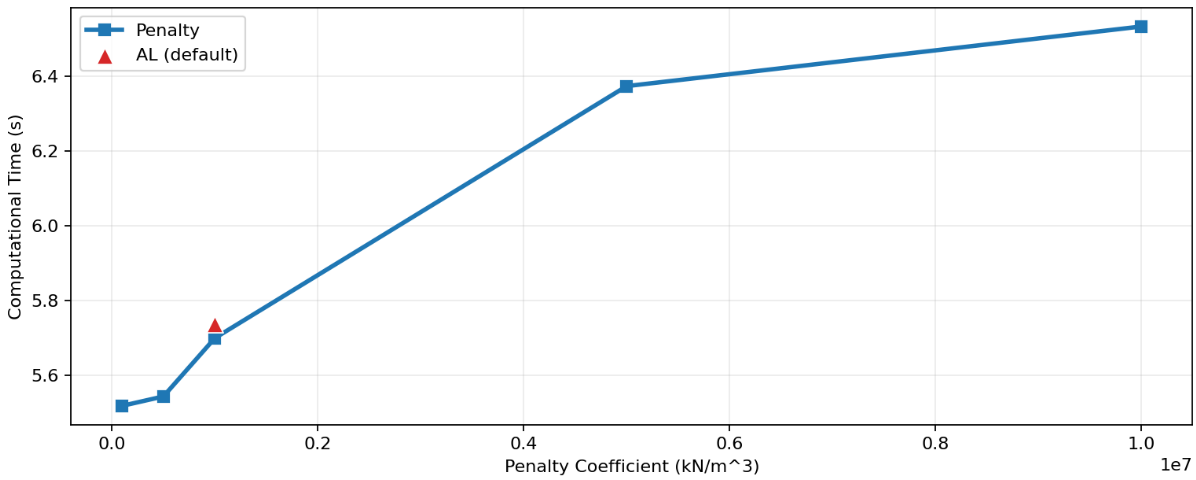

Figure 2: Analysis time vs penalty coefficient

2.0.2 Problem Description

A smooth, rigid circular cylinder of diameter $D = 1.2\,\mathrm{m}$ (radius $R = 0.6\,\mathrm{m}$) and weight $W = 350\,\mathrm{kg}$ is placed in contact with an elastic half-space under its own weight. By symmetry only half the cylinder is modelled. Contact is enforced via a friction-free penalty formulation under load control.

2.0.3 Model Setup

- **Cylinder**: rigid, circular
- **Radius**: $R = 0.6$ m
- **Weight**: $W = 350$ kg (line load $P = W\,g = 350 \times 9.81 = 3433.5$ N/m)
- **Soil**: linear elastic, $E = 100$ MPa, $\nu = 0.3$
- **Contact**
  - Normal penalty: k_n (varied—see below)
  - Friction coefficient: $\mu = 0$

2.0.4 Simulation Input Snippet

Rigid-Motion Constraint This block applies a prescribed vertical force (line load) to the rigid cylinder during Step 2 only, using static force-control (regulation) to match the target reaction:

- **Constraint ID**: 2
- **MotionType**: Translation (no rotation)

- **RigidBodyID**: 1 (the cylinder)
- **ForceY**: Prescribed vertical force (compression negative); ramped in Step 2
- **ForceTolerance / ForceRegMaxIters / ForceRegGain**: Force-control regulation settings
- **StepIds**: 2 (activate in Step 2; in Step 1 the cylinder is simply positioned / brought into contact)

```
% RigidMotionConstraints
@RigidMotionConstraint 2
@@MotionType: Translation
@@RigidBodyID: 1
@@ForceY: -3.4335
@@ForceLoadY: LoadType Ramp Step 2
@@ForcePropagateStepsY: 2
@@ForceTolerance: 1.0e-3
@@ForceRegMaxIters: 50
@@ForceRegGain: 1e-4
@@StepIds: 2
%%% static force-control (regulation) in Step 2
```

Step Definitions (Step 2) Below, Step 2 applies a static analysis with a direct solver:

- **SolverType**: Direct
- **StartStep** = 1 (first increment)
- **StepTime** = 1.0 (total pseudo-time)
- **NumberSteps** = 1000 (increments)
- **OutputInterval** = 10 (write every 10 increments)
- **ErrorTarget** = 1e-1, **MaxIterations** = 100 (nonlinear solver tolerances)
- **OutputTypes**: Displacement, EffStress
- **UL**: No (no Updated Lagrangian)
- **SimMode**: Static

```
% Step Definitions for static loading
@Step 2:
@@SolverType: Direct
@@StartStep: 1
@@StepTime: 1.0
@@NumberSteps: 1000
@@OutputInterval: 10
@@ErrorTarget: 1.e-1
@@MaxIterations: 100
@@OutputTypes: Displacement EffStress
@@UL: No
@@SimMode: Static
```

2.0.5 Analytical Solution

For plane-strain contact of an infinitely stiff (rigid) cylinder under line load P on an elastic half-space:

- **Equivalent modulus**

$$E^* = \frac{E}{1-\nu^2} = \frac{100 \times 10^6 \text{ Pa}}{1-0.3^2} \approx 1.10 \times 10^8 \text{ Pa} \tag{2}$$

- **Contact half-width**

$$a = \sqrt{\frac{4PR}{\pi E^*}} = \sqrt{\frac{4 \times 3433.5 \text{ N/m} \times 0.6 \text{ m}}{\pi \times 1.10 \times 10^8 \text{ Pa}}} \approx 4.89 \times 10^{-3} \text{ m} \tag{3}$$

- **Peak contact pressure**

$$p_0 = \frac{2P}{\pi a} = \frac{2 \times 3433.5 \text{ N/m}}{\pi \times 4.89 \times 10^{-3} \text{ m}} \approx 4.52 \times 10^5 \text{ Pa} = 0.452 \text{ MPa} \tag{4}$$

Penalty-Parameter Study

k_n (kN/m³)	p_0 (MPa)	Comment
1×10^5	0.13	Penalty too small; excessive penetration
1×10^6	0.26	Under-stiff penalty; peak $\ll$ analytical
2×10^6	0.32	Under-stiff penalty; underestimates p_0 by ~29%; solver stable
5×10^6	0.40	Under-stiff penalty; underestimates p_0 by ~11%; solver stable
7×10^6	0.43	Under-stiff penalty; underestimates p_0 by ~4%; solver stable
1×10^7	0.46	Near-analytical peak; solver stable

The strip footing tolerates a broader patch and displacement control better; the cylinder's narrow contact under load control is most sensitive.

2.1 Example 3: Rigid Body Contact with Displacement Control Followed by Force Control

2.1.1 Input File Name

`fem_data.txt`

2.1.2 Problem Description

A rigid body is initially positioned 1 cm above the slave surface. The analysis consists of two steps: first, a displacement-controlled step lowers the rigid body to establish contact, followed by a force-controlled step that applies a prescribed force to reach static equilibrium.

2.1.3 Model Setup

- **Rigid Body**: Initially positioned 1 cm above the slave surface
- **Soil**: Linear elastic, $E = 100$ MPa, $\nu = 0.02$
- **Contact**:
  - Normal penalty: $k_n = 5 \times 10^5$ kN/m^3
  - Tangential penalty: $k_t = 5 \times 10^5$ kN/m^3
  - Friction coefficient: $\mu = 0$ (frictionless)
- **Loading**:
  - **Step 1**: Displacement control—rigid body lowered by $\delta = 0.99$ cm (0.0099 m)
  - **Step 2**: Force control—applied force $F_y = -20$ N (downward) to reach static equilibrium

2.1.4 Step Definitions

Step 1: Displacement-Controlled Lowering Step 1 applies a static, displacement-controlled analysis to lower the rigid body and establish initial contact:

- **SolverType**: Direct
- **SimMode**: Static
- **StartStep**: 0
- **StepTime**: 1.0
- **InitialStepIncrement**: 0.1
- **ErrorTarget**: 1.0e-1
- **MaxIterations**: 100
- **OutputTypes**: Displacement, TotalStress

```
% Step Definitions
@Step 1:
@@ModernAutoInc: Yes
@@SolverType: Direct
@@StartStep: 0
@@StepTime: 1
@@MaxIterations: 100
```

```
@@OutputInterval: 1
@@InitialStepIncrement: 1e-1
@@UseModifiedNewton: No
@@OutputTypes: Displacement TotalStress
@@Geostatic: No
@@MinTimeStep: 1e-7
@@MaxTimeStep: 1.0e0
@@MaxRetryCount: 5
@@ErrorTarget: 1.0e-1
@@FnormDamping: 0.0
@@UL: No
@@SimMode: Static
%%%
```

Step 2: Force-Controlled Equilibrium Step 2 applies a static, force-controlled analysis to reach equilibrium under the prescribed force:

- **SolverType**: Direct
- **SimMode**: Static
- **StartStep**: 1
- **StepTime**: 1.0
- **InitialStepIncrement**: 0.001
- **ErrorTarget**: 1.0e-1
- **MaxIterations**: 100
- **OutputTypes**: Displacement, TotalStress

```
% Step Definitions
@Step 2:
@@ModernAutoInc: Yes
@@SolverType: Direct
@@StartStep: 1
@@StepTime: 1
@@MaxIterations: 100
@@OutputInterval: 1
@@InitialStepIncrement: 1e-3
@@UseModifiedNewton: No
@@OutputTypes: Displacement TotalStress
@@Geostatic: No
@@MinTimeStep: 1e-7
@@MaxTimeStep: 1.0e0
@@MaxRetryCount: 5
@@ErrorTarget: 1.0e-1
```

```
@@FnormDamping: 0.0
@@UL: No
@@SimMode: Static
%%%
```

2.1.5 Rigid Motion Constraints

Constraint 1: Displacement Control (Step 1) This constraint prescribes a constant downward velocity to lower the rigid body in Step 1:

- **Constraint ID**: 1
- **MotionType**: Translation
- **RigidBodyID**: 1
- **VelEqY**: Defines vertical velocity as $v_y(t) = a + b\,t + c\,t^2 + \ldots$. Here only $a = -0.0099$ m/s is nonzero, so the rigid body moves downward at 0.99 cm/s, resulting in a total displacement of 0.99 cm over the step duration.
- **StepIds**: 1

```
% RigidMotionConstraints
@RigidMotionConstraint 1
@@MotionType: Translation
@@RigidBodyID: 1
@@VelEqY: a=-0.0099 b=0 c=0 d=0 f=0 g=0
@@StepIds: 1
```

Constraint 2: Force Control (Step 2) This constraint applies a prescribed force to the rigid body in Step 2 to reach equilibrium:

- **Constraint ID**: 2
- **MotionType**: Translation
- **RigidBodyID**: 1
- **ForceY**: -20.0 N (downward force)
- **ForceLoadY**: LoadType Ramp Step 2 (ramped loading)
- **ForcePropagateStepsY**: 2
- **ForceTolerance**: 1.0 N
- **ForceRegMaxIters**: 5
- **ForceRegGain**: 1e-6
- **StepIds**: 2

```
@RigidMotionConstraint 2
@@MotionType: Translation
@@RigidBodyID: 1
@@ForceY: -20.0
@@ForceLoadY: LoadType Ramp Step 2
@@ForcePropagateStepsY: 2
@@ForceTolerance: 1.0e0
@@ForceRegMaxIters: 5
@@ForceRegGain: 1e-6
@@StepIds: 2
%%%
```

2.1.6 Contact Pair Definition

The contact pair defines the interaction between the rigid body (master) and the soil surface (slave):

```
% Contact Pairs
@ContactPair 1
@@MasterNodes [rigid body node IDs]
@@SlaveNodes [slave surface node IDs]
@@OrderOfContact: 2
@@NumGaussPoints: 30
@@PenaltyCoefficientNormal: 5e5
@@PenaltyCoefficientTraction: 5e5
@@Friction: 0.0
@@TLOUTS: 0.0
@@TLOPEN: 0.0
@@InitiationStepId: 1
%%%
```

2.1.7 Analytical Solution

For a rigid strip footing on an elastic half-space, the settlement and contact pressure can be determined using analytical solutions.

Settlement (Giroud's Solution) For a rectangular foundation on a soil layer, Giroud (1972) provides a settlement factor P_{Hm} that depends on Poisson's ratio ν and the depth-to-width ratio H/B. For a strip footing with $H/B = 10$:

- $P_{Hm}(\nu = 0) = 2.177$
- $P_{Hm}(\nu = 0.3) = 1.894$

For $\nu = 0.02$, linear interpolation yields:

$$P_{Hm}(0.02) = 2.177 + \frac{0.02}{0.3}(1.894 - 2.177) = 2.177 + 0.0667(-0.283) \approx 2.158 \quad (5)$$

The settlement under a uniform pressure p is given by:

$$w = \frac{pB}{E} P_{Hm} \quad (6)$$

For the applied force $F = 20\ \mathrm{kN/m}$ on a strip of width $B = 1\ \mathrm{m}$:

- Average pressure: $p = F/B = 20\ \mathrm{kPa}$
- Young's modulus: $E = 100\ \mathrm{MPa} = 100,000\ \mathrm{kPa}$
- Settlement factor: $P_{Hm} = 2.158$

$$\frac{pB}{E} = \frac{20 \times 1}{100,000} = 0.0002\ \mathrm{m}$$

$$w = 0.0002 \times 2.158 = 0.0004316\ \mathrm{m} = 0.432\ \mathrm{mm} \quad (7)$$

The predicted settlement is approximately **0.43 mm** downward.

Additionally, the peak contact pressure at the center is approximately **12.73 kPa** per the analytical contact solution.

2.1.8 Penalty-Parameter Study

A series of numerical simulations was performed by varying the normal penalty coefficient k_n to assess its effect on settlement and contact pressure predictions. The results are compared against the analytical solution (settlement: 0.43 mm, center pressure: 12.73 kPa).

k_n (kN/m³)	Settlement (mm)	Center Pressure (kPa)	Comment
1×10^5	0.6	—	Excessive penetration; penalty too small
1×10^6	0.41	13.2	Good agreement with analytical solution; stable convergence

k_n (kN/m³)	Settlement (mm)	Center Pressure (kPa)	Comment
5×10^6	—	—	Oscillatory solutions; solver instability

The penalty coefficient of $k_n = 1 \times 10^6$ kN/m^3 provides the best balance between accuracy and numerical stability. The predicted settlement (0.41 mm) closely matches the analytical value (0.43 mm), with a relative error of approximately 4.7%. The center contact pressure (13.2 kPa) is also in good agreement with the analytical prediction (12.73 kPa), with a relative error of about 3.7%.

At the lower penalty value ($k_n = 1 \times 10^5$ kN/m^3), the penalty is insufficient to prevent excessive penetration, resulting in an overestimated settlement of 0.6 mm (40% higher than analytical). At the higher penalty value ($k_n = 5 \times 10^6$ kN/m^3), the system becomes numerically stiff, leading to oscillatory behavior and convergence difficulties.

2.1.9 Analysis Sequence

1. **Initial Configuration**: The rigid body is positioned 1 cm above the slave surface with no contact forces.
2. **Step 1 (Displacement Control)**: The rigid body is lowered by 0.99 cm at a constant velocity, establishing contact with the slave surface. This displacement-controlled step ensures controlled contact initiation.
3. **Step 2 (Force Control)**: A downward force of 20 N is applied to the rigid body using a ramped loading scheme. The force-controlled step allows the system to reach static equilibrium under the prescribed load.

This two-step approach demonstrates the transition from displacement control (useful for establishing contact) to force control (useful for equilibrium analysis under prescribed loads).

2.2 Example 4: Adding Friction and Tangential Motion

2.2.1 Input File Name

`fem_data_friction.txt`

2.2.2 Input Extension

Example 3 can be extended by adding a third step and activating friction along the contact interface. The same rigid body and soil are used, but the contact pair is now assigned a finite friction coefficient:

- **Contact** (modified):

- Normal penalty: $k_n = 1.0 \times 10^6 \text{ kN/m}^3$
- Tangential penalty: $k_t = 1.0 \times 10^6 \text{ kN/m}^3$
- Friction coefficient: $\mu = 0.2$

On the rigid-body side we keep the two original constraints from Example 3 and add a third one:

- **Constraint 1 (Step 1)** – downward velocity in Y (same as before).
- **Constraint 2 (Step 2)** – downward force $F_y = -20 \text{ kN}$ (interpreting the nominal "20" as 20 kN per unit thickness).
- **Constraint 3 (Step 3)** – a prescribed horizontal motion in X:

```
@RigidMotionConstraint 3
@@MotionType: Translation
@@RigidBodyID: 1
@@VelEqX: a=-0.01 b=0 c=0 d=0 f=0 g=0
@@StepIds: 3
%%%
```

Step 3 is defined as another static step (same solver and time stepping as Step 2), but now the contact is frictional and the rigid body is driven tangentially along the interface.

2.2.3 From Normal Loading to Frictional Resistance

By the end of Step 2 (vertical force control), the reaction recorded on the rigid body in the **vertical** direction is approximately

$$R_y \approx -19.2 \text{ kN}, \tag{11}$$

which is consistent with the target -20 kN once contact compliance and penalty effects are taken into account. This vertical reaction becomes the "normal force" available to generate frictional resistance in Step 3.

With $\mu = 0.2$, the classical Coulomb law gives a limit tangential (sliding) resistance of

$$R_{x,\max} = \mu |R_y| \approx 0.2 \times 19.2 \approx 3.8 \text{ kN}. \tag{12}$$

Equivalently, in terms of a friction angle φ,

$$\mu = \tan\varphi \implies \varphi = \arctan(0.2) \approx 11.3^\circ, \tag{13}$$

so the contact can sustain shear forces up to about 3.8 kN before sliding.

2.2.4 What the Simulation Reports in Step 3

In the Step-3 output file, the rigid body reaction in the X direction (column `RbRx`) settles at

$$R_x \approx 3.84 \text{ kN}, \tag{14}$$

while the vertical reaction remains close to $R_y \approx -19.2$ kN. This is almost exactly

$$R_x \approx \mu |R_y| \approx 0.2 \times 19.2 \text{ kN}, \tag{15}$$

showing that the numerical solution is sitting right on the Coulomb friction envelope: the rigid body is trying to move horizontally, the contact enforces stick up to the limit $R_{x,\max}$, and beyond that the interface yields and slides with a reaction locked at the friction capacity.

In other words, Example 4 turns the purely normal contact of Example 3 into a **frictional contact test**:

- Steps 1–2 establish a normal load of roughly 20 kN.
- Step 3 introduces a tangential drive in X.
- The recorded $R_x \approx 3.84$ kN is precisely the shear force that a contact with $\mu = 0.2$ can mobilize under that normal load.

This makes Example 4 a compact demonstration that:

- The rigid-body/contact formulation correctly accumulates rigid-body reactions.
- The tangential traction is capped by $\mu|R_n|$ in line with Coulomb friction.
- The `MasterForceContact` post-processing and rigid-body reaction logging can be used to verify frictional contact behavior quantitatively.

2.3 References

1. Budynas, R., & Nisbett, K. (2008). *Shigley's Mechanical Engineering Design* (8th ed.). McGraw-Hill.
2. Beer, F. P., & Johnston, E. R. (1992). *Mechanics of Materials* (2nd ed.). McGraw-Hill.
3. Giroud, J. P. (1972). Settlement of rectangular foundation on soil layer. *Journal of the Soil Mechanics and Foundations Division*, 98(1), 149-154.