AD FALCON API Manual

Coupled Dynamic Analysis with FALCON (Saturated Soils)

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Contents

1 Coupled Dynamic Analysis with FALCON (Saturated Soils)

1.1 Introduction

This reference summarises the governing equations for saturated-soil dynamics—mass, momentum, and energy balances coupled with Darcy flow and Terzaghi's effective stress.
All symbols used below are listed once in **Table 1**.

1.2 Conservation of Mass

For saturated conditions, only two phases are present: solid $\alpha = s$ and water $\alpha = w$.

For each phase:

$$\frac{\partial(\rho_\alpha n^\alpha)}{\partial t} + \nabla \cdot \left(\rho_\alpha n^\alpha \dot{\mathbf{u}}_\alpha\right) = 0 \tag{1}$$

1.2.1 Solid phase

With $\dot{\mathbf{u}}_s = \dot{\mathbf{u}}$ and Darcy velocity $\dot{\mathbf{w}}^w$:

$$\frac{Dn}{Dt} = (1 - n)\nabla \cdot \dot{\mathbf{u}} \tag{2}$$

Assumptions leading to Eq. (2): Solid grains are assumed incompressible with spatially uniform density ($\nabla \rho_s = 0$). The material time derivative $D(\cdot)/Dt = \partial(\cdot)/\partial t + \dot{\mathbf{u}} \cdot \nabla(\cdot)$ is applied to the solid-phase mass balance.

1.2.2 Water phase

$$\frac{n}{K_w}\frac{Dp_w}{Dt} + \nabla \cdot \dot{\mathbf{u}} + \nabla \cdot \dot{\mathbf{w}}^w + \dot{\mathbf{w}}^w \cdot \frac{\nabla \rho_w}{\rho_w} = 0 \tag{3}$$

Assumption for isothermal compressibility: In an isothermal environment, the rate of change in fluid density is related to pressure through the bulk modulus:

$$\frac{1}{\rho_w}\frac{D\rho_w}{Dt} = \frac{1}{K_w}\frac{Dp_w}{Dt}$$

where K_w is the bulk modulus of water.

1.3 Weak Forms

1.3.1 Fluid mass balance

$$\int_\Omega \delta p_w A_w \, d\Omega + \int_{\Gamma_{qw}} \delta p_w(\dot{\mathbf{w}}^w \cdot \mathbf{n}^* - \overline{\dot{\mathbf{w}}}^w) \, d\Gamma = 0 \tag{13}$$

with

$$A_w = \frac{n}{K_w}\frac{Dp_w}{Dt} + \nabla \cdot \dot{\mathbf{u}} + \nabla \cdot \dot{\mathbf{w}}^w + \dot{\mathbf{w}}^w \cdot \frac{\nabla \rho_w}{\rho_w} \tag{14}$$

1.3.2 Generalized Darcy law (after Zienkiewicz et al. (1990); Lewis & Schrefler (1999))

$$\dot{\mathbf{w}}^w = \mathbf{k}_w[-\nabla p_w + \rho_w(\mathbf{b} - \ddot{\mathbf{u}})], \qquad \mathbf{k}_w = \mathbf{k}_{\text{int}}/\eta_w \tag{15}$$

where $\mathbf{k}_{\text{int}}$ is the intrinsic permeability tensor and η_w is the water viscosity. **Assumption:** Relative fluid acceleration $\ddot{\mathbf{u}}_{ws}$ is neglected.

1.3.3 Solid momentum balance

$$\int_\Omega \delta\mathbf{u}^\top(\mathbf{L}^\top\boldsymbol{\sigma} + \rho\mathbf{b} - \rho\ddot{\mathbf{u}})\,d\Omega + \int_{\Gamma_t} \delta\mathbf{u}^\top(\bar{\mathbf{t}} - \mathbf{I}_\sigma^\top\boldsymbol{\sigma})\,d\Gamma = 0 \tag{16}$$

1.4 FE Discrete Equations

Unknown vectors: $\mathbf{U}, \mathbf{P}_w$.

Kinematic framework: Small-strain formulation is used for the stiffness matrix unless geometric nonlinearity (updated Lagrangian formulation) is explicitly enabled for large-deformation problems.

$$\mathbf{M}_u\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} - \mathbf{Q}_w\mathbf{P}_w = \mathbf{F}_u \tag{17}$$

$$\mathbf{M}_w\ddot{\mathbf{U}} + \mathbf{Q}_w^\top\dot{\mathbf{U}} + \mathbf{C}_{ww}\dot{\mathbf{P}}_w + \mathbf{H}_{ww}\mathbf{P}_w = \mathbf{F}_w \tag{18}$$

Note on the damping matrix $\mathbf{C}$:
For coupled analyses, the global damping matrix $\mathbf{C}$ in Eq. (17) comprises both material damping (proportional to density and a damping coefficient in dynamic analyses) and velocity-pressure coupling contributions from $\mathbf{Q}_w^\top$.

(See Appendix A for detailed matrix definitions.)

1.4.1 Reduced System: Negligible Fluid-Phase Inertia ($\mathbf{M}_w \approx \mathbf{0}$)

Key assumption: In many geomechanics applications the inertia of pore water is negligible compared with the solid skeleton. Assuming $\mathbf{M}_w \approx \mathbf{0}$, the discrete equations reduce to

$$\mathbf{M}_u\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} - \mathbf{Q}_w\mathbf{P}_w = \mathbf{F}_u \tag{17R}$$

$$\mathbf{Q}_w^\top\dot{\mathbf{U}} + \mathbf{C}_{ww}\dot{\mathbf{P}}_w + \mathbf{H}_{ww}\mathbf{P}_w = \mathbf{F}_w \tag{18R}$$

These reduced equations are used by default in FALCON unless the inclusion of fluid-phase inertia is explicitly required.

1.4.2 Special Case: Consolidation Analysis

For **consolidation analysis** (quasi-static coupled analysis), all acceleration terms vanish ($\ddot{\mathbf{U}} = \mathbf{0}$), and the discrete equations reduce to:

$$\mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} - \mathbf{Q}_w\mathbf{P}_w = \mathbf{F}_u \tag{17C}$$

$$\mathbf{Q}_w^\top\dot{\mathbf{U}} + \mathbf{C}_{ww}\dot{\mathbf{P}}_w + \mathbf{H}_{ww}\mathbf{P}_w = \mathbf{F}_w \tag{18C}$$

This formulation represents **Terzaghi's consolidation theory** in its general form, capturing the time-dependent dissipation of excess pore pressure and associated deformation of the soil skeleton. The coupling between solid deformation rate ($\dot{\mathbf{U}}$) and pore pressure ($\mathbf{P}_w$) governs the consolidation process.

Note: In many consolidation problems, the damping term $\mathbf{C}\dot{\mathbf{U}}$ in Eq. (17C) is neglected, further simplifying to:

$$\mathbf{K}\mathbf{U} - \mathbf{Q}_w\mathbf{P}_w = \mathbf{F}_u \tag{17C$'$}$$

1.5 Appendix A: Finite-Element Matrices and Vectors

1.5.1 Mass matrices

$$\mathbf{M}_u = \int_\Omega \mathbf{N}_u^\top \rho\, \mathbf{N}_u \, d\Omega \tag{19}$$

$$\mathbf{M}_w = \int_\Omega (\nabla\mathbf{N}_{pw})^\top \mathbf{k}_w \rho_w \, \mathbf{N}_u \, d\Omega \tag{20}$$

1.5.2 Coupling matrices

$$\mathbf{Q}_w = \int_\Omega \mathbf{B}^\top \mathbf{m}\, \mathbf{N}_{pw} \, d\Omega \tag{21}$$

1.5.3 Hydraulic conductivity matrix

$$\mathbf{H}_{ww} = \int_\Omega (\nabla\mathbf{N}_{pw})^\top \mathbf{k}_w \nabla\mathbf{N}_{pw} \, d\Omega \tag{23}$$

1.5.4 Compressibility matrix

$$\mathbf{C}_{ww} = \int_\Omega \mathbf{N}_{pw}^\top \mathbf{C}_1^* \mathbf{N}_{pw} \, d\Omega \tag{24}$$

with

$$\mathrm{C}_1^* = \frac{n}{K_w} \tag{25}$$

1.5.5 Load and flow vectors

$$\mathbf{F}_u = \int_\Omega \mathbf{N}_u^\top \rho\, \mathbf{b}\, d\Omega + \int_{\Gamma_t} \mathbf{N}_u^\top \bar{\mathbf{t}}\, d\Gamma \tag{26}$$

$$\mathbf{F}_w = \int_\Omega (\nabla \mathbf{N}_{pw})^\top \mathbf{k}_w \rho_w \mathbf{b}\, d\Omega - \int_{\Gamma_{qw}} \mathbf{N}_{pw}^\top \bar{\bar{\mathbf{w}}}^w\, d\Gamma \tag{27}$$

1.6 Table 1: Variable Definitions

Symbol	Description
$M^\alpha,\ \Omega^\alpha$	mass, volume of phase $\alpha \in \{s, w\}$
$\rho_\alpha,\ \rho^\alpha,\ \rho$	phase density, partial density, mixture density
$n^\alpha,\ n$	phase volume fraction, porosity
p_w	pore-water pressure
$\boldsymbol{\sigma},\ \boldsymbol{\sigma}'$	total stress, effective stress (Terzaghi)
$K_w,\ \eta_w$	bulk modulus and viscosity of water
$\mathbf{k}_{\text{int}},\ \mathbf{k}_w$	intrinsic and effective permeability
$\mathbf{u},\ \dot{\mathbf{u}},\ \ddot{\mathbf{u}}$	displacement, velocity, acceleration (solid)
$\mathbf{w}^w$	Darcy velocity of water phase
ε	strain tensor
$\mathbf{b}$	body-force vector
$E_I,\ E_k,\ \mathcal{P}$	internal energy, kinetic energy, power
$\mathrm{N},\ \mathrm{B},\ \mathrm{L}$	shape, strain–displacement, differential operators
$\mathrm{M},\ \mathrm{K},\ \mathrm{C},\ \mathrm{H}$	global mass, stiffness, damping/coupling, conductivity matrices
$\mathbf{Q}_w$	solid–fluid coupling matrix
$\mathbf{M}_u,\ \mathbf{M}_w$	phase-assembled mass matrices
$\mathbf{H}_{ww}$	hydraulic conductivity matrix
$\mathbf{C}_{ww}$	compressibility matrix
C_1^*	element-level coefficient (see Appendix A)
$\mathbf{n}^*,\ \mathrm{I}_\sigma^\top$	outward unit normal, stress-traction operator
$\bar{\mathbf{t}}$	prescribed traction (Neumann BC)
$\bar{\bar{\mathbf{w}}}^w$	prescribed Darcy flux of water phase
$\mathrm{U},\ \mathrm{P}_w$	global DOF vectors (displacement, p_w)
$\mathbf{F}_u,\ \mathbf{F}_w$	global load / flow vectors
$\mathbf{m}$	Coupling vector $= [1,\ 1,\ 1,\ 0,\ 0,\ 0]^\top$ in 3D Voigt notation

1.7 References

O. C. Zienkiewicz, Y. Xie, B. Schrefler, *et al.* (1990) *Static and dynamic behaviour of soils: a rational approach to quantitative solutions. I. Fully saturated problems. Proc. Royal Soc. A* 429, 285–309.

Lewis, R.W. & Schrefler, B.A. (1999). *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media*. 2nd ed. Wiley.