AD FALCON API Manual

Unified Clay and Sand Model (CASM)

Javad Ghorbani

March 26, 2026

Contents

1 Unified Clay and Sand Model (CASM)

CASM is a **critical-state model** (Yu, 1998) that unifies clay-like and sand-like responses by combining:

- A flexible yield surface controlled by the exponent n
- Rowe-type stress–dilatancy for **non-associated** flow
- A single isotropic hardening variable controlling the cap size

This implementation is **saturated-only**; for unsaturated analyses see the GCC model.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/CASMModelUMAT.cpp path/to/CASMModelUMAT.hpp Mechanical \
    Phi=30 Lambda=0.15 Kappa=0.03 Nu=0.25 Alpha=0.8 SSC=2 SPR=1.2 \
    P_min=0.1 DefaultIsoHardening=500 v_N=2.0 STOL=1e-5 FTOL=1e-6 LTOL=1e-6
\
    CustomVariable=IsotropicHardening
```

For readability, this example is wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

Use the parameter names shown in the tables below.

Notes:

- This UMAT expects exactly the required parameter set shown in the table; additional `name=value` pairs are not supported.
- Optional `OCR` is provided as a custom state variable (not as a UMAT parameter).

1.2 Material parameters

Table 1. Material parameters and their descriptions

Symbol	Keyword in input	Units	Required	Description
ϕ'	`Phi`	°	✓	Critical-state friction angle.

Symbol	Keyword in input	Units	Required	Description
λ	`Lambda`	–	✓	Virgin compression index (slope of NCL in v–$\ln p$ space). Must satisfy $\lambda > \kappa$.
κ	`Kappa`	–	✓	Swelling/reloading index. Must be positive and less than λ.
ν	`Nu`	–	✓	Poisson's ratio. Must satisfy $-1 < \nu < 0.5$.
α	`Alpha`	–	✓	Deviatoric shape parameter used in F_θ.
n	`SSC`	–	✓	Yield exponent controlling shape (CASM notation n).
r	`SPR`	–	✓	Spacing ratio (input must satisfy **SPR > 1**). Internally $r^* = 1/\ln(r)$.
v_N	`v_N`	–	✓	Specific volume at $p = 1$ (same convention as the code).
$P_{\min}$	`P_min`	stress	✓	Lower bound used inside elastic moduli to prevent $K, G \to 0$ at very small p. Must be positive.
a_0	`DefaultIso Hardening`	stress	✓	Additive floor used by post-equilibrium conditioning (see below). Must be positive.

Symbol	Keyword in input	Units	Required	Description
`STOL`	`STOL`	–	✓	Substepping/integration tolerance.
`FTOL`	`FTOL`	–	✓	Yield-surface tolerance used for branching and drift correction.
`LTOL`	`LTOL`	–	✓	Load–unload detection tolerance.

Notes

- **No `OCRControlled` and no `OverBurdenPressure`** parameters are used by the current CASM UMAT (unlike the unsaturated GCC model).
- **SPR validation**: because the code uses $r^* = 1/\ln(r)$, the input must satisfy $r > 1$ to avoid division by zero or negative values.

1.2.1 Constructor validation

The UMAT constructor performs the following checks and throws `std::invalid_argument` if any fails:

Condition	Rationale
$\kappa > 0$	Ensures positive elastic stiffness.
$\lambda > \kappa$	Required for physically meaningful plastic hardening.
$P_{\min} > 0$	Prevents zero/negative modulus floor.
$-1 < \nu < 0.5$	Valid Poisson's ratio bounds.
$a_0 > 0$ (`DefaultIsoHardening`)	Prevents unphysical zero cap floor.
$r > 1$ (SPR)	Ensures $\ln(r) > 0$ so r^* is finite and positive.

1.3 Custom state variables

Name	Required	Meaning
`Isotropic Hardening`	✓	The saturated cap size variable used by CASM (denoted σ_{mc0} in the code, equivalent to p_0 in p–q space).

Name	Required	Meaning
OCR	×	Optional multiplier used only during **post-equilibrium conditioning** (acts like b_0). Default is 0.

1.4 Elastic law

The UMAT uses pressure-dependent isotropic elasticity with stress-dependent tangent moduli:

$$K = v\,\frac{\max(P_{\min},\, p)}{\kappa}, \qquad v = 1 + e \tag{1}$$

$$G = \frac{3\,(1-2\nu)}{2\,(1+\nu)}\,K \tag{2}$$

where $p = -\sigma_m$ is the compressive mean pressure, e is the void ratio and $P_{\min} > 0$ is a numerical lower bound that prevents unbounded compliance at very small stresses.

The elastic stiffness matrix is

$$\mathbf{D}^e = \begin{bmatrix} \lambda_e + 2\mu & \lambda_e & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e + 2\mu & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e & \lambda_e + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \lambda_e = K - \tfrac{2}{3}G,\ \mu = G \tag{3}$$

1.5 Yield surface

The CASM yield function (in compressive $p > 0$ form) is:

$$f = \left(\frac{q\,F_\theta}{M\,p}\right)^n + r^*\,\ln\left(\frac{p}{p_0}\right) = 0 \tag{4}$$

where:

- M is the critical state slope: $M = \frac{6\sin\phi'}{3-\sin\phi'}$
- p_0 is the isotropic hardening/cap variable (stored as `IsotropicHardening`)
- n is the yield exponent (SSC)
- $r^* = 1/\ln(r)$ with $r =$ SPR

The Lode-angle factor is

$$F_\theta = \frac{1}{\alpha}\left[\frac{1+\alpha^4-(1-\alpha^4)R_\theta}{2}\right]^{1/4}, \qquad R_\theta = -\frac{3\sqrt{3}\,J_3}{2J_2^{3/2}} \tag{5}$$

1.6 Plastic potential

CASM uses **non-associated flow** based on Rowe-type stress–dilatancy. The implementation follows the UMAT's internal equations for $\partial g/\partial\sigma$ and returns the plastic direction through `GradientOfPlasticToSigma()`.

1.7 Hardening law

The cap variable grows with plastic volumetric strain:

$$p_0 = p_{00}\,\exp\left(\frac{\varepsilon_v^p}{\lambda-\kappa}\right) \tag{6}$$

The plastic modulus K_p is computed from the chain rule:

$$K_p = -\frac{\partial f}{\partial p_0}\cdot B_{\mathrm{iso}}, \qquad B_{\mathrm{iso}} = \frac{\partial g}{\partial p}\frac{\partial p_0}{\partial \varepsilon_v^p} \tag{7}$$

1.8 Elastoplastic tangent

Loading on the yield surface enforces the consistency condition

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} : d\boldsymbol{\sigma} + \frac{\partial f}{\partial p_0}\,dp_0 = 0 \tag{8}$$

yielding the plastic multiplier

$$d\lambda = \frac{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e d\boldsymbol{\varepsilon}}{K_p + \dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e : \dfrac{\partial g}{\partial \boldsymbol{\sigma}}} \tag{9}$$

The elastoplastic tangent is

$$\mathbf{D}_{ep} = \mathbf{D}^e - \frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}} \otimes \frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{D}^e}{K_p + \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e : \frac{\partial g}{\partial \boldsymbol{\sigma}}} \tag{10}$$

1.9 Post-equilibrium conditioning (saturated)

After a geostatic/equilibrium step, the host code may have established a stress state (p, q, θ) and a void ratio e, but the stored hardening variable may not be consistent with the current stress point and the CASM yield surface.

The UMAT's `setCustomVariable()` performs a **post-equilibrium conditioning** procedure that mirrors the "a_0, b_0" idea described in the GCC documentation (see gccmodel.md), but without any unsaturation/hydraulic coupling. This ensures that after initialization, the stress state lies either on or inside the yield surface.

1.9.1 Step 1 — Fit the minimum cap size through the current stress

From the current stress invariants, compute

- $p = -\sigma_m$ (compression-positive)
- q (equivalent deviatoric stress)
- F_θ (Lode-angle factor)

Then compute the **minimum** cap size $p_{0,\min}$ that makes the yield function pass exactly through the current stress state (i.e., $f = 0$):

$$p_{0,\min} = \frac{p}{\exp\left(-\frac{1}{r^*}\left(\frac{qF_\theta}{Mp}\right)^n\right)} \tag{11}$$

In the code, p is locally bounded using `P_min` (only for log/division safety) to avoid numerical issues if the mean stress is extremely small. This local bound does **not** affect the deviatoric stress invariants.

1.9.2 Step 2 — Apply a_0 and b_0 (via `DefaultIsoHardening` and `OCR`)

The UMAT then initializes the stored hardening variable (`IsotropicHardening`) as:

$$p_{0,\text{init}} = \max\left(p_{0,\min},\ a_0 + b_0\, p_{0,\min}\right) \tag{12}$$

with

- $a_0 =$ `DefaultIsoHardening`
- $b_0 =$ `OCR` (optional; defaults to 0)

So:

- If you set `OCR = 0`, then $p_{0,\text{init}} = \max(p_{0,\min}, a_0)$.
- Larger `OCR` increases the initialized cap size proportionally to $p_{0,\min}$, pushing the stress point further inside the yield surface (higher apparent overconsolidation).

1.9.3 Step 3 — Update void ratio to be consistent with the initialized cap

Finally, the void ratio is updated using the elastic unloading branch of the NCL:

$$v = v_N + \kappa \left(\ln(p_{0,\text{init}}) - \ln(p)\right) - \lambda \ln(p_{0,\text{init}}), \qquad e = v - 1 \tag{13}$$

This ensures that (p, e, p_0) are mutually consistent with the elastoplastic framework at the start of loading.

1.10 Numerical integration

The UMAT uses **adaptive substepping** with error control governed by `STOL`. Each substep applies:

1. **Elastic trial** — compute trial stress assuming purely elastic response.
2. **Yield check** — evaluate f at the trial stress; if $f \le$ FTOL, accept elastic.
3. **Load–unload detection** — use `LTOL` and a regula-falsi (Illinois) intersection finder to locate the yield-surface crossing within the substep.
4. **Plastic return** — enforce consistency via the implicit return-mapping algorithm.
5. **Drift correction** — iteratively project the stress back onto the yield surface if numerical drift exceeds `FTOL`.

Denominator guards and numerical safeguards are applied throughout to prevent division-by-zero and NaN propagation.

1.11 Example input (triaxial driver)

```
# --- CASM driver input (units: stress in kPa; compression negative) ---
Mode  Drained

nSteps     2000
dEpsAxial  -1e-4

# Material parameters
Phi   23
Lambda 0.093
Nu     0.30
Kappa  0.025
Alpha  0.78
SSC    4.5
SPR    2.714
P_min  0.1
DefaultIsoHardening 207.5
v_N    2.1071
```

```
# Optional: post-equilibrium multiplier
# OCR 0.0

# Solver tolerances
STOL 1e-7
FTOL 1e-4
LTOL 1e-6

# Initial state
VoidRatio 0.632
StressXX  -207
StressYY  -207
StressZZ  -207

# Custom state (if omitted, post-equilibrium conditioning will set it)
# IsotropicHardening 207.5
```

1.12 Verification: Drained triaxial tests on Weald Clay

This section provides a compact drained-triaxial verification for the CASM implementation using a Weald Clay parameter set under two overconsolidation conditions. The purpose is to offer a repeatable single-point check of stress–strain and volumetric trends under the same drained triaxial loading constraint used by the packaged mini.

1.12.1 Test conditions and parameters

The same CASM material parameters are used for both tests.

Material parameters (Weald Clay)

Symbol	Keyword	Value
ϕ'	`Phi`	23.0°
λ	`Lambda`	0.093
κ	`Kappa`	0.025
ν	`Nu`	0.30
v_N	`v_N`	2.1071
α	`Alpha`	0.78
n	`SSC`	4.5
r	`SPR`	2.714

Initial conditions

The initial stress is isotropic with mean effective stress p_0, applied in the driver as $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p_0$ (compression is negative). The initial cap size is provided as $p_{c0} = \mathrm{OCR} \cdot p_0$ through `IsotropicHardening`.

Test	OCR	Initial void ratio e_0	p_0 (kPa)
Normally consolidated	1	0.632	207
Heavily overconsolidated	24	0.617	34.5

1.12.2 Input files (packaged mini cases)

These verification inputs are included as packaged CASM mini cases:

- OCR = 1: `mini_tools/CASM/cases/drained_weald_clay_ocr1/input.txt`
- OCR = 24: `mini_tools/CASM/cases/drained_weald_clay_ocr24/input.txt`

Copies of the same mini-driver inputs are also provided under:

- `Falcon_inputs/casm/weald_clay_drained_ocr1.txt`
- `Falcon_inputs/casm/weald_clay_drained_ocr24.txt`

1.12.3 Results

Key trends reproduced by these two single-point simulations:

- Normally consolidated case: contractive volumetric response under drained loading.
- Heavily overconsolidated case: peak deviatoric strength followed by post-peak softening, with a dilative volumetric response at larger strains.

1.13 FALCON mini

The packaged mini tool id is `CASM`. It lives under `mini_tools/CASM`.

1.13.1 How to run

```
falcon --mini-root /path/to/UMATLIB_FALCON/falcon_minis --mini-tool CASM
--mini-input /path/to/UMATLIB_FALCON/falcon_minis/CASM/cases/drained
```

Packaged simulation families:

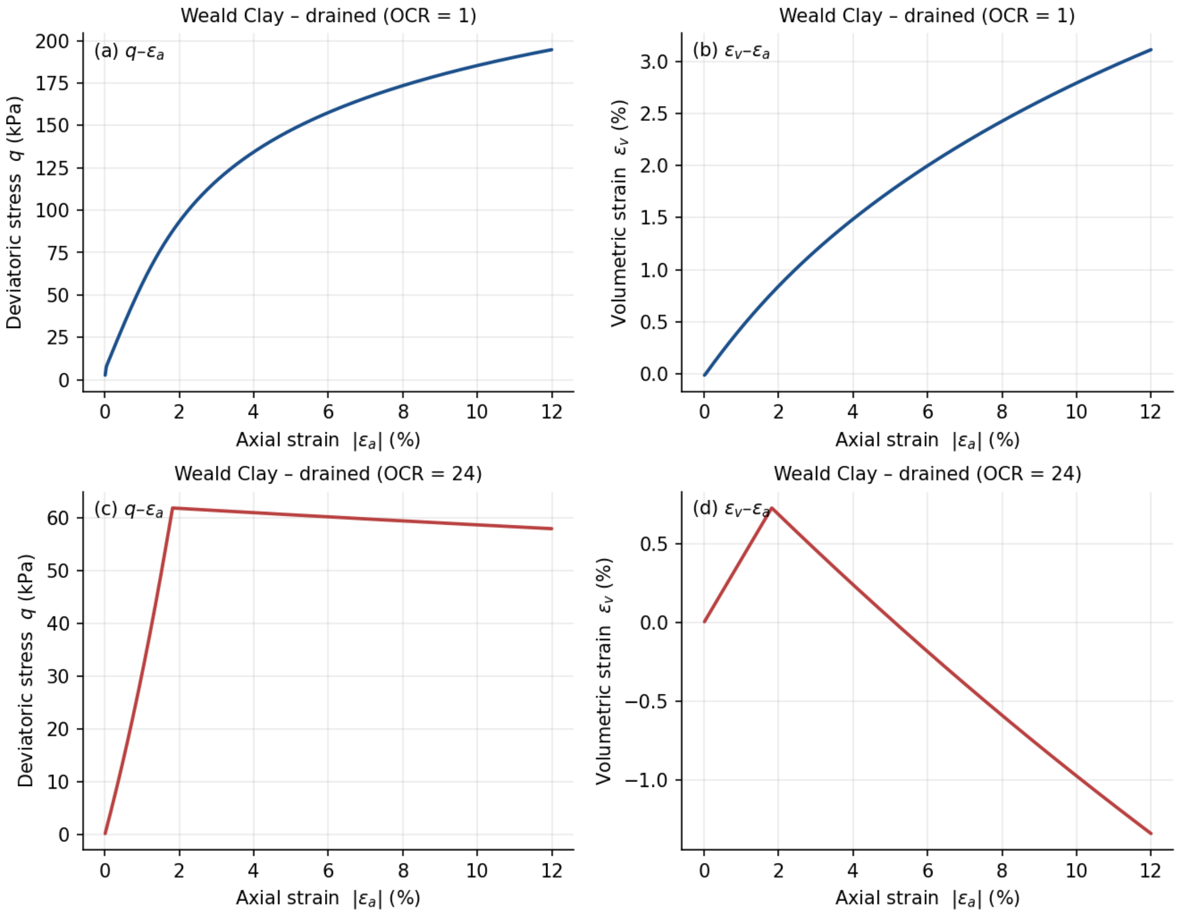


Figure 1: Weald Clay drained verification (OCR 1 and 24)

Packaged case	Path	Purpose
Drained triaxial	cases/drained/input.txt	Saturated drained reference path.
Undrained triaxial	cases/undrained/input.txt	Saturated constant-volume reference path.

1.13.2 Input syntax

`input.txt` uses whitespace-delimited `Key Value` pairs, one item per line, for example:

```
Mode Drained
Phi 23.0
Lambda 0.25
StressXX -100.0
```

The main driver selector is `Mode`. For the packaged CASM mini cases, each input file contains:

- one loading-program selector (`Mode`)
- one CASM constitutive parameter block
- one initial stress and state block
- one state-initialization control block
- one loading-control block

`Mode` value	Meaning in the standalone mini	Mechanical constraint
`Drained`	Saturated drained triaxial loading. The driver applies axial strain and solves the radial strain increment so the lateral total stress remains close to the target confining stress.	Constant-confining triaxial compression.
`Undrained`	Saturated constant-volume triaxial loading. The driver applies axial strain with `exx = ezz = -0.5 eyy`.	Zero total volumetric strain.

Mini inputs used by the packaged cases:
Constitutive parameters:

Input key	Used by	Required / choices / defaults	Meaning
`Phi`	all cases	Required in packaged cases	Critical-state friction angle defining the CASM stress ratio `M`.
`Lambda`	all cases	Required in packaged cases	Virgin compression slope of the normal compression line.
`Kappa`	all cases	Required in packaged cases	Elastic swelling/reloading slope.
`Nu`	all cases	Required in packaged cases	Elastic Poisson ratio.
`Alpha`	all cases	Required in packaged cases	Lode-angle dependence parameter in the CASM yield surface.
`SSC`	all cases	Required in packaged cases	CASM yield exponent `n`, controlling the shape of the surface in `p-q` space.
`SPR`	all cases	Required in packaged cases	Spacing ratio `r`, used internally through `r^* = 1 / ln(r)`.
`P_min`	all cases	Optional; pressure-floor default exists but packaged cases set it explicitly	Lower pressure floor used in the elastic tangent.
`DefaultIso Hardening`	all cases	Optional; packaged cases set it explicitly	Additive hardening floor used when the state is auto-initialized from the starting stress point.
`v_N`	all cases	Required in packaged cases	Specific-volume intercept of the normal compression line at `p = 1`.

Input key	Used by	Required / choices / defaults	Meaning
STOL, FTOL, LTOL	all cases	Optional; packaged cases set them explicitly	Stress-integration, drift, and load-detection tolerances used by the driver and UMAT.

Initial stress and state inputs:

Input key	Used by	Required / choices / defaults	Meaning
StressXX, StressYY, StressZZ	all cases	Required in packaged cases	Initial total stress components. The packaged cases use an isotropic starting stress state.
VoidRatio	all cases	Required in packaged cases	Initial void ratio supplied to the standalone driver.
IsotropicHardening	all cases	Required if AutoInitializeState = 0; packaged cases still provide it	Starting CASM hardening size if raw values are used directly.
OCR	all cases	Required when auto-initializing state; packaged cases set it explicitly	Overconsolidation-style multiplier used during automatic initialization of the state.
AutoInitializeState	all cases	Optional; choices 0 / 1; packaged cases use 1	If 1, the driver rebuilds the hardening size and void ratio from the current stress state and OCR; if 0, it uses the raw VoidRatio and IsotropicHardening values directly.

Loading controls:

Input key	Used by	Required / choices / defaults	Meaning
`dEpsAxial`	all cases	Required in packaged cases	Imposed axial strain increment. Compression is negative in the packaged CASM mini drivers.
`nSteps`	all cases	Required in packaged cases	Number of accepted loading increments.
`OutputCSV`	all cases	Optional; defaults to `stress_results.csv` when omitted in packaged usage	Optional output file name written in the case directory.

1.13.3 Hydromechanical assumptions

The packaged CASM mini is saturated only:

- no suction, retention, or degree-of-saturation update is used
- the stress variable of interest is the saturated effective mean stress `p`
- all state evolution comes from the CASM mechanical hardening law and the chosen triaxial boundary condition

That makes the CASM mini much simpler than GCC or the unsaturated Mohr mini. The key modeling choice here is not hydraulic coupling but how the state is initialized:

- if `AutoInitializeState = 1`, the driver reconstructs a CASM-consistent starting state from the stress point and `OCR`
- if `AutoInitializeState = 0`, the input file is treated as a manually prescribed raw state

The packaged cases use `AutoInitializeState = 1`, which is usually the safer option for single-point verification runs.

1.13.4 Sample input

Drained triaxial example Path: `mini_tools/CASM/cases/drained/input.txt`

```
Mode Drained

Phi 23.0
Lambda 0.25
Nu 0.30
```

```
Kappa 0.05
Alpha 0.78
SSC 2.5
SPR 10.0
P_min 0.1
DefaultIsoHardening 50.0
v_N 2.6
STOL 1e-3
FTOL 1e-4
LTOL 1e-6

StressXX -100.0
StressYY -100.0
StressZZ -100.0
VoidRatio 0.60
IsotropicHardening 50.0
OCR 2.0
AutoInitializeState 1

dEpsAxial -1e-4
nSteps 400
OutputCSV stress_results.csv
```

This is the saturated drained reference path for the CASM mini. The constitutive parameters are kept identical to the undrained case, so the main difference between the two packaged examples is only the loading constraint, not the material definition.

Undrained triaxial example Path: `mini_tools/CASM/cases/undrained/input.txt`

```
Mode Undrained

Phi 23.0
Lambda 0.25
Nu 0.30
Kappa 0.05
Alpha 0.78
SSC 2.5
SPR 10.0
P_min 0.1
DefaultIsoHardening 50.0
v_N 2.6
STOL 1e-3
FTOL 1e-4
```

```
LTOL 1e-6

StressXX -100.0
StressYY -100.0
StressZZ -100.0
VoidRatio 0.60
IsotropicHardening 50.0
OCR 2.0
AutoInitializeState 1

dEpsAxial -1e-4
nSteps 400
OutputCSV stress_results.csv
```

This packaged case uses the same CASM parameter set and the same initialized state as the drained example but switches the driver to the constant-volume branch. It is therefore a clean companion case for comparing drained and undrained CASM stress paths from the same starting point.

1.13.5 Output files and columns

Each run writes `stress_results.csv`.

Output file	Produced by	Main use
`stress_results.csv`	all cases	Main mechanical history used by the packaged CASM figures.

Primary output columns in `stress_results.csv`:

Output column	Meaning
`step`	Load-step index written by the standalone driver.
`exx`, `eyy`, `ezz`, `ezy`, `ezx`, `exy`	Strain components for the accepted step state.
`sxx`, `syy`, `szz`, `szy`, `szx`, `sxy`	Stress components for the accepted step state.
`q`, `p`	Deviatoric and mean stress invariants used by the packaged CASM plots.
`e`	Void ratio.
`isotropic_hardening`	Current CASM hardening size carried by the UMAT.

When reading the packaged CASM outputs, a practical workflow is:

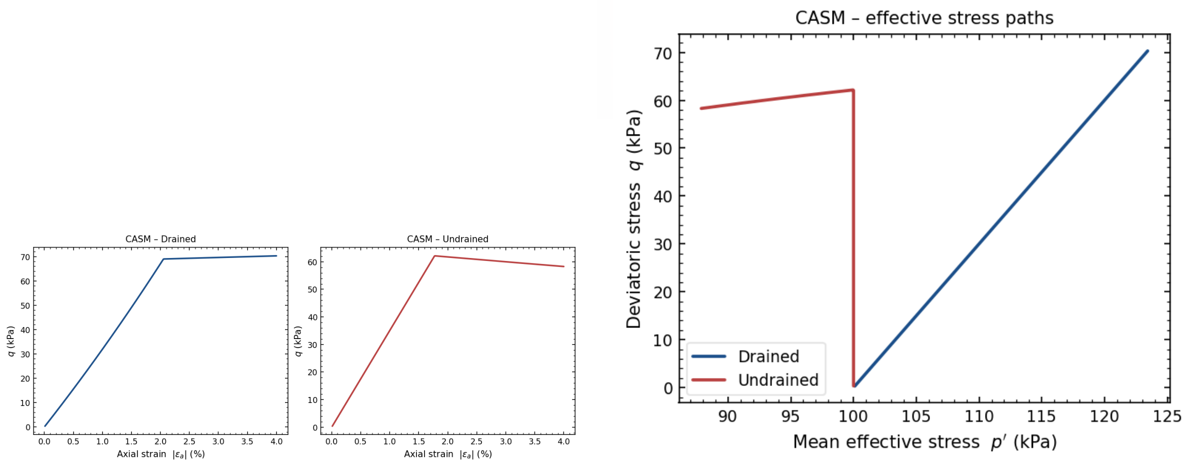
1. inspect `q` and `p` first to understand the triaxial stress path
2. inspect `e` next if you want to see how the drained path changes volume
3. inspect `isotropic_hardening` if you want to follow cap growth during loading

The plots in the next section are generated from these packaged case CSVs.

1.14 Results

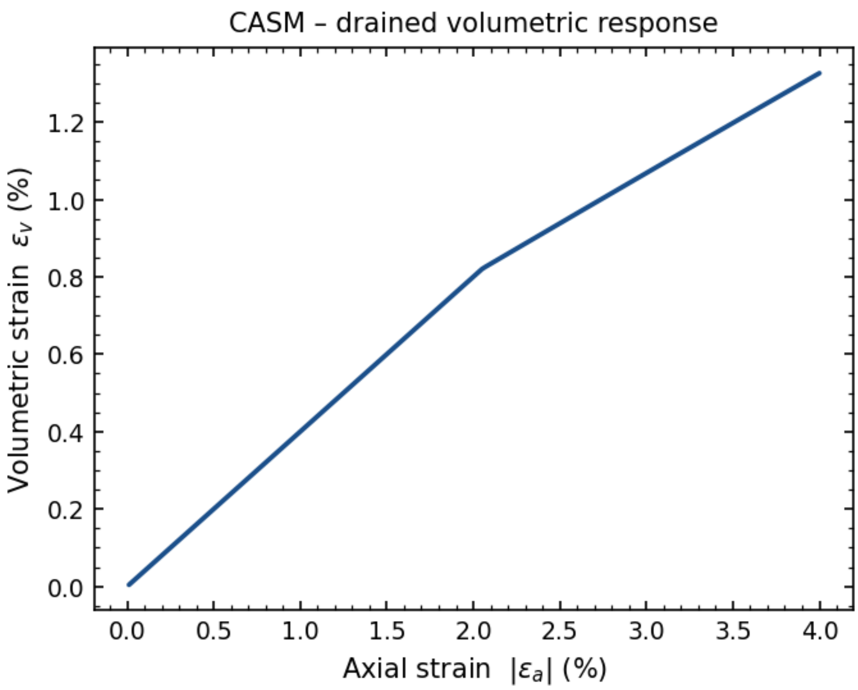
The plots below are produced directly from the bundled FALCON mini case inputs under `mini_tools/CASM/cases`. They show how the same CASM parameter set responds under drained and undrained triaxial constraints after the state has been auto-initialized from the common isotropic starting stress.

1.14.1 Drained and undrained triaxial comparison



Bundled cases `cases/drained/input.txt` and `cases/undrained/input.txt`. Left: deviatoric response `q-\varepsilon_a` for the two packaged triaxial paths. Right: the corresponding `q-p` stress paths. This paired view is the most direct way to see how the same initialized CASM state responds differently once lateral drainage is either permitted or blocked by the loading constraint.

1.14.2 Drained volumetric response



Bundled case `cases/drained/input.txt`. This figure isolates the drained volumetric-strain response (volume change) from the same packaged case. For CASM, this is one of the most informative single-output checks because the model is designed to capture clay-like and sand-like responses through the interaction between stress ratio, dilatancy, and hardening rather than through suction coupling.

1.15 Applications and limitations

- Best suited to critical-state effective-stress soil behavior spanning clay-like and sand-like responses within one unified framework.
- Appropriate for uncoupled and effective-stress-based coupled analyses where suction-specific hardening is not required inside the constitutive law.
- This formulation is saturated only and is not a cyclic mobility or liquefaction model.

1.16 References

- Yu, H. S. (1998). *CASM: a unified state parameter model for clay and sand.* International Journal for Numerical and Analytical Methods in Geomechanics, 22(8), 621–653.