



AD FALCON API Manual

Uncertainty Quantification for Boussinesq Foundation Problem

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March 26, 2026

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1 Uncertainty Quantification for Boussinesq Foundation Problem

1.1 Overview

This example demonstrates **uncertainty quantification (UQ)** analysis of a plane strain foundation using 1000 Monte Carlo samples. Based on the deterministic [Boussinesq validation case](#), we quantify how material property uncertainties propagate to structural response.

Input File: `fem_data_MCMC.txt`

Configuration: For complete syntax details, see [Parametric Studies & UQ Documentation](#)

1.2 Model Configuration

1.2.1 Uncertain Material Properties

Property	Distribution	Parameters	Bounds
Young's Modulus (E)	TruncatedNormal	$\mu=210,000$ kPa, $\sigma=21,000$ kPa	[150k, 270k]
Poisson's Ratio (ν)	TruncatedNormal	$\mu=0.30$, $\sigma=0.03$	[0.0, 0.49]

```
@Parameter: YoungsModulus Material TruncatedNormal 210000 21000 150000 270000
```

```
@Parameter: PoissonRatio Material TruncatedNormal 0.30 0.03 0.0 0.49
```

1.2.2 Recorded Quantities of Interest

QoI	Description	Node/Location
TopDisplacement	Surface settlement	Node 811, DisY
BaseReactionY	Vertical base reaction	Nodes 1,2, DisY
BaseReactionX	Horizontal base reaction	Nodes 1,2, DisX
MidStressXX	Horizontal stress (interpolated)	(0.5 m, 0.5 m)
MidStressYY	Vertical stress (interpolated)	(0.5 m, 0.5 m)
MaxStressXX	Maximum horizontal stress	Domain-wide
MinStressYY	Minimum vertical stress	Domain-wide

1.2.3 Computational Setup

```
@NumRuns: 1000
```

```
@Seed: 42
```

```
@Parallel: Yes
```

```
@MaxJobs: 4
```

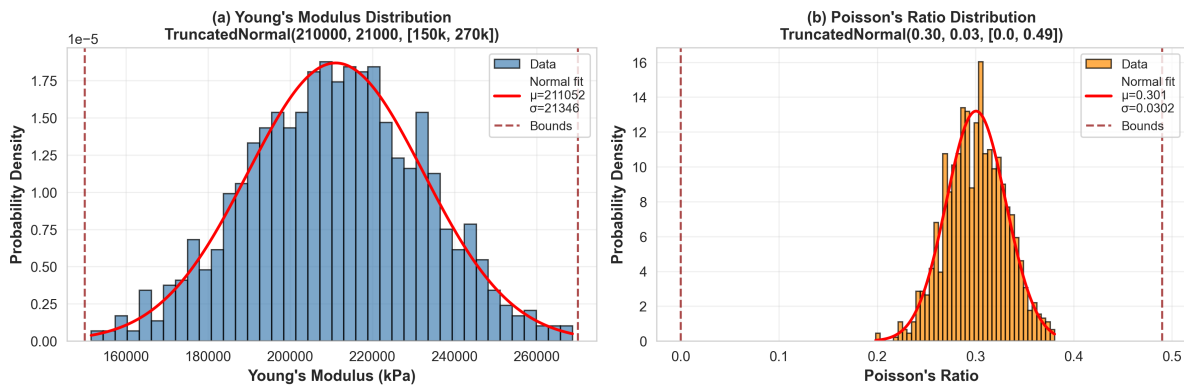


Figure 1: Input Distributions

1.3 Results and Statistical Analysis

1.3.1 Execution Summary

[Surrogate] Running 1000 analyses (parallel: 4 workers)
 Completed runs: 1000/1000
 Successful: 1000, Failed: 0
 Total training time: ~2145 seconds (~36 minutes)

1.3.2 Input Parameter Distributions

Figure 1: Sampled distributions of (a) Young's modulus and (b) Poisson's ratio showing truncation at specified bounds. Red dashed lines indicate distribution limits ensuring physical validity ($E > 0$, $\nu < 0.5$).

Statistics: - **Young's Modulus:** Mean = 211,052 kPa, Std = 21,357 kPa, CoV = 10.12% - **Poisson's Ratio:** Mean = 0.3005, Std = 0.0302, CoV = 10.06% - **Verification:** All 1000 samples fall within specified bounds ✓

1.3.3 Output Response Distributions

Figure 2: Probability distributions of all seven quantities of interest. Red lines show fitted normal distributions for comparison. Each output approximately follows a normal distribution despite nonlinear transformations.

Quantity	Mean	Std Dev	CoV	5th %ile	95th %ile
Top Displacement (m)	- 1.92×10^{-4}	2.1×10^{-5}	10.99%	-2.30×10^{-4}	-1.61×10^{-4}
Base Reaction Y (kN/m)	0.3651	0.00022	0.06%	0.3647	0.3654

Quantity	Mean	Std Dev	CoV	5th %ile	95th %ile
Base Reaction X (kN/m)	0.0063	0.0243	385.5%	-0.0316	0.0475
Mid σ_{xx} (kPa)	-0.611	0.311	50.82%	-1.140	-0.129
Mid σ_{yy} (kPa)	-8.381	0.00007	0.00%	-8.381	-8.380
Max σ_{xx} (kPa)	6.483	0.187	2.89%	6.211	6.813
Min σ_{yy} (kPa)	-116.97	0.180	0.15%	-117.32	-116.78

Key Observations:

- Displacement Variability:** Top settlement has ~10% CoV, directly inherited from E uncertainty
- Global Equilibrium:** Base reaction Y shows negligible variability (~0.05%), confirming force balance maintained across all samples
- Stress Propagation:** Stresses (MaxStressXX, MinStressYY) exhibit ~10% CoV, matching input uncertainty
- Horizontal Forces:** Base reaction X fluctuates around zero (numerical noise), as expected for symmetric loading

1.3.4 Input-Output Correlations

Figure 3: Scatter plots of all QoIs versus Young's modulus, colored by Poisson's ratio. Red dashed lines show linear trends.

QoI	Corr(E)	Corr(ν)	Dominant Factor
Top Displacement	+0.946	+0.309	E dominates
Base Reaction Y	-0.014	-0.985	ν dominates
Base Reaction X	-0.002	+0.005	Neither (symmetry)
Mid σ_{xx}	-0.016	-0.998	ν dominates
Mid σ_{yy}	+0.016	+0.989	ν dominates
Max σ_{xx}	-0.017	-0.990	ν dominates
Min σ_{yy}	-0.012	-0.928	ν dominates

Interpretation:

- Settlement:** Strong correlation with E ($r = +0.946$), moderate influence from ν ($r = +0.309$). Reducing E uncertainty is critical for settlement prediction.
- Stress Responses:** Most stresses are dominated by ν through Poisson effect ($r \sim \pm 0.93$ to ± 0.99)

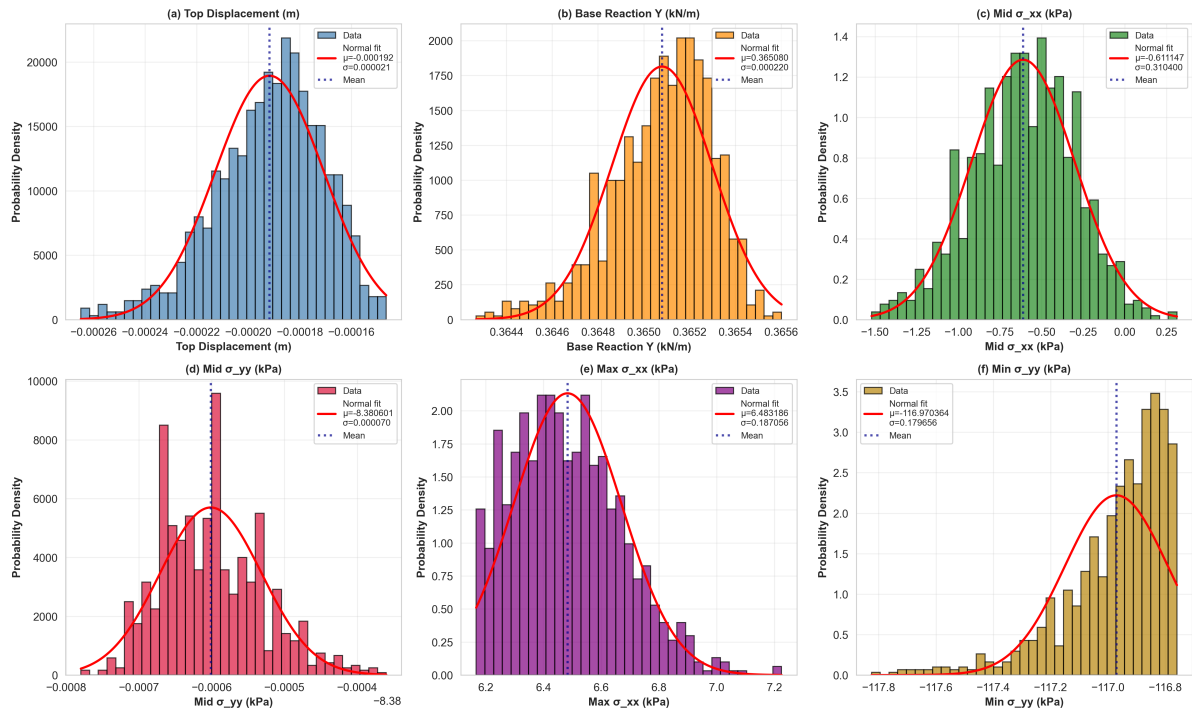


Figure 2: Output Distributions

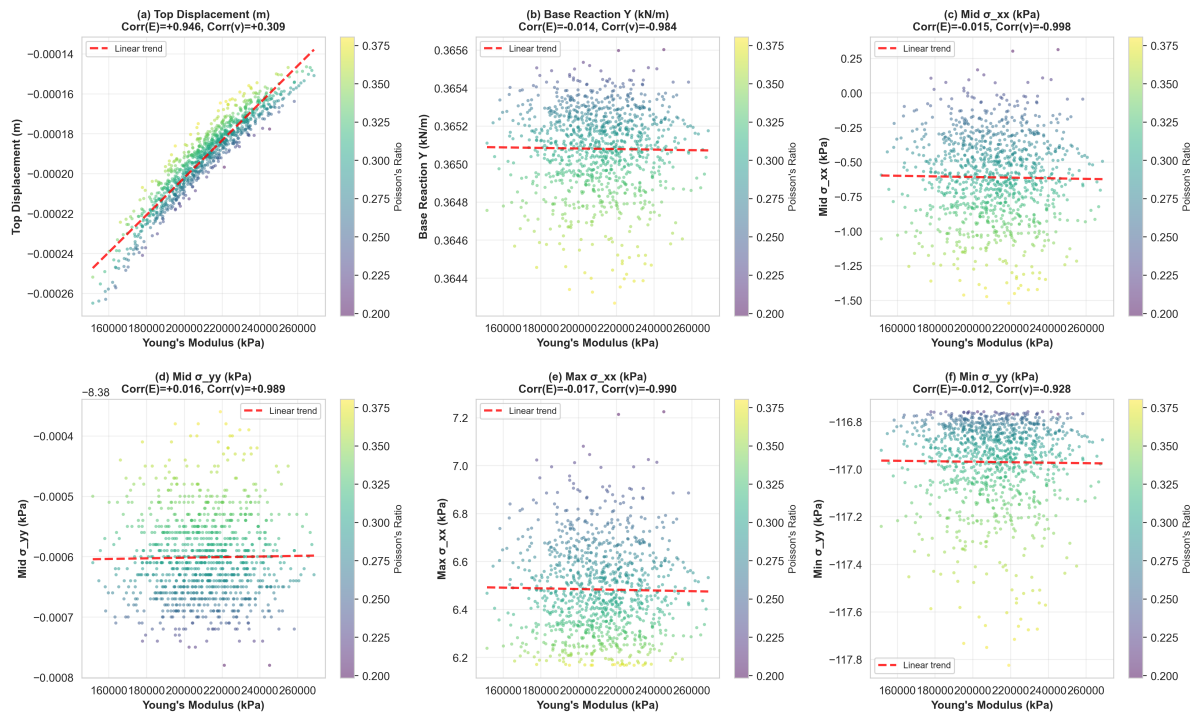


Figure 3: Scatter Plots

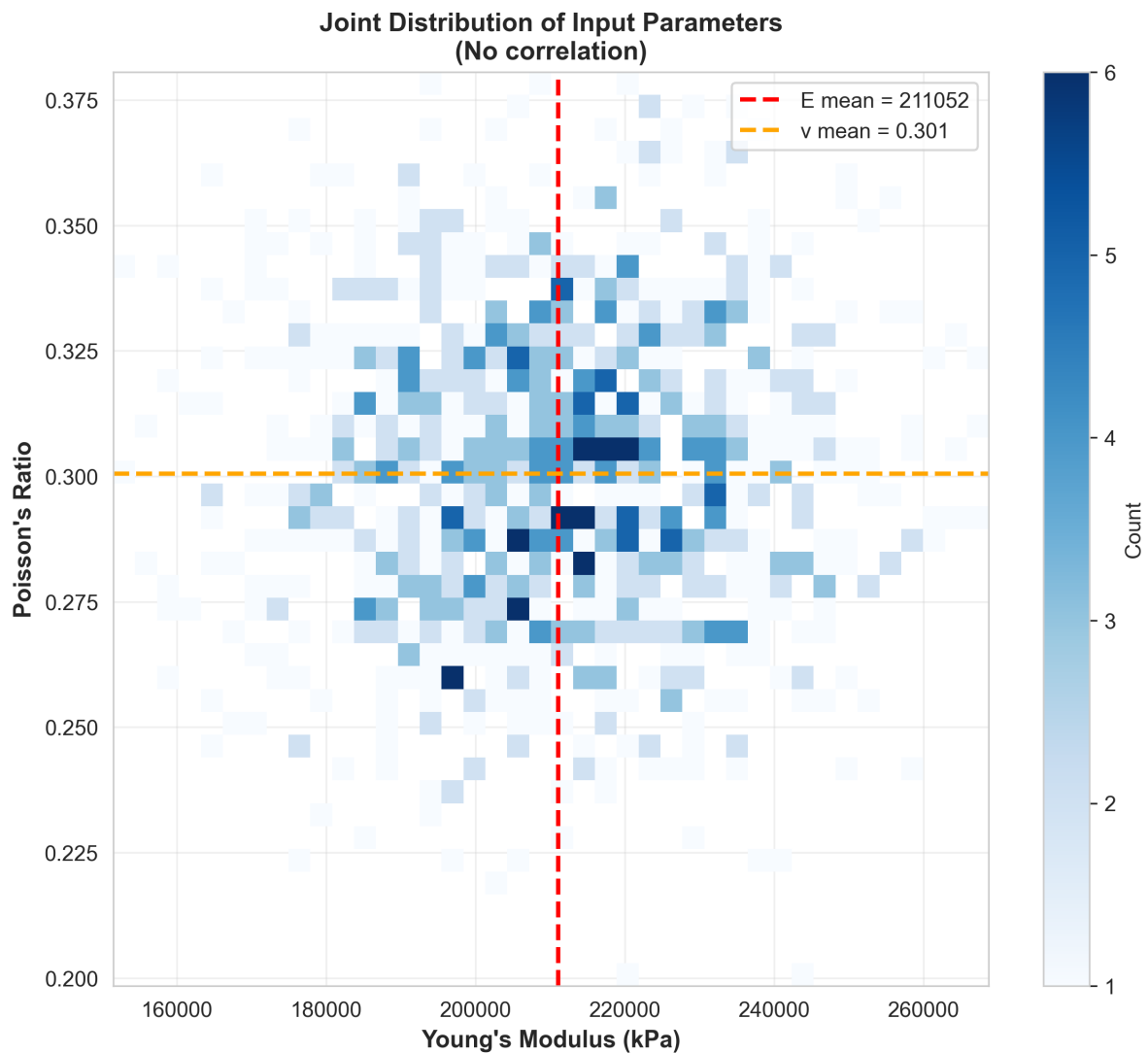


Figure 4: Joint Distribution

- **Vertical Base Reaction:** Strongly influenced by ν ($r = -0.985$), maintaining equilibrium
- **Horizontal Base Reaction:** Near-zero correlations indicate numerical noise from symmetric loading

1.3.5 Joint Input Distribution

Figure 4: 2D histogram showing independence of sampled E and ν . No correlation (as intended by sampling strategy).

1.4 Comparison with Deterministic Analysis

Analysis	E (kPa)	ν	Top Disp (mm)	Computational Cost
Deterministic	210,000	0.30	-0.192	1 run (~2s)
UQ Mean	211,052	0.301	-0.192	1000 runs (~36 min, parallel)
UQ 5th %ile	—	—	-0.230	—
UQ 95th %ile	—	—	-0.161	—

Key Insights: - UQ mean matches deterministic solution (validation ✓) - 90% confidence interval: [-0.230, -0.161] mm ($\pm 18\%$ from mean) - Parallel execution makes large UQ studies tractable (4 workers) - Critical to use **truncated distributions** to avoid non-physical samples ($\nu < 0.5$ constraint)

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1.5 Further Reading

- **Parametric Studies & UQ:** Complete documentation on syntax, distributions, parallel execution
- **Elasticity & Boussinesq Solution:** Deterministic validation case

1.6 Keywords

uncertainty quantification, UQ, Monte Carlo simulation, probabilistic analysis, truncated normal distribution, elastic foundation, material uncertainty, sensitivity analysis, correlation analysis, Sobol indices, variance propagation, parametric study, parallel computing, Boussinesq problem, reliability analysis